PARETO OPTIMIZATION DESIGN TECHNIQUES FOR THE AFIT/AAMRL ANTHROPOMORPHIC ROBOTIC MANIPULATOR

THESIS

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Abstract

A method to optimize a robotic parallel manipulator configuration using Pareto Optimization techniques was developed. Pareto optimization is a cooperative effort between design parameters. The design parameters to be optimized included the payload mass, the length of the manipulator link labelled l_2, and the prescribed time for the manipulator to move a prescribed distance. Three functionals were computed for design optimization. These included the mechanical efficiency of the system, the maximum value of torque for motor one, and the maximum value of torque for motor two.

A functional analysis was performed using two trajectories for the manipulator; a horizontal trajectory and a vertical trajectory. A combination of these paths allow the manipulator to reach anywhere within its workspace.

Algorithms were developed for computing each of the functionals when changing any of the design parameters. When the horizontal path was traversed, mechanical efficiency was zero, thus total input work of the manipulator was evaluated. The sensitivities of the design parameter changes were evaluated for optimization. When a horizontal path was followed, only the link l_2 length had changing sensitivity values. Sensitivity changes occurred for all of the design parameters for a vertical trajectory.

Pareto Optimization Design Techniques For the AFIT/AAMRL Anthropomorphic Robotic Manipulator

I. Introduction

The Air Force Institute of Technology (AFIT), in conjunction with the Armstrong Aeronautical Medical Research Laboratory (AAMRL), has identified the need to purchase or fabricate an anthropomorphic robotic manipulator (A^3RM) for future robotics research projects in order to study human telepresence. In a telepresence application, an individual straps on an exoskeleton and movement of this exoskeleton causes the same movements by the robot arm. This removes the individual from a potential hostile or hazardous environment. Previous research studied several different configurations available for a manipulator [9,3-1]. A numerical analysis of optimization techniques on the robotic manipulator parallel configuration will be accomplished and a software tool will be developed for finding sensitivity changes caused by changing manipulator design parameters.

1.1 Background

More computing power is currently available than ever before due to the technological advances of the past few years. Engineers have better computer aided design tools and new numerical techniques to solve extremely complicated design problems in the optimization field.

We shall present optimization parameters that are of interest to a robotics designer, along with the usual design constraints. Available optimization algorithms will be discussed along with values of safety factors involved in optimization solutions.

Previously, some research has been conducted into comparison of different robotic manipulator configurations. Captain Steven L. Parker compared serial and parallel manipulator configurations. Each of these configurations had two degrees of freedom. Based on an efficiency analysis, he concluded that the parallel manipulator configuration is superior to the serial configuration [9:3-1]. However, Captain Parker did not investigate requirements concerning optimizing of either of these designs in accordance with any predefined constraints.

1.2 Performance Requirements

According to Kanoh [3:1866], robot performance requirements include high speed operation, accurate positioning, accurate tracking, light weight, reduced inertia, more maneuverability, and minimal energy consumption. High speed performance depends on the power output of the motors that drives the robot manipulator. Therefore, it is necessary to know the size of the motors that will be used for the manipulator or at least a power

range. Accurate positioning is defined as the capability of the manipulator to move from the starting point to the ending point within a predetermined range. Accurate tracking is the ability of the manipulator to follow an object.

It is possible to optimize other parameters than those mentioned in the preceding paragraph. One of these parameters is the efficiency over the whole workspace. Efficiency can be defined several ways : (1) the least amount of movement by the manipulator, (2) the least amount of energy consumed by the manipulator, or (3) the least amount of error correction needed. It is possible to minimize the variation in dynamics; that is, to make the system decoupled and configuration invariant, gravity balanced, and so forth. Weight ratio rather than light weight can be investigated for design improvements. Weight ratio is the mass of the arm compared to the maximum payload mass. Other possible optimization parameters include maximum speed and maximum rate of acceleration of the manipulator.

1.3 Constraints

Normally, manufacturing or technological limitations impose constraints on values of the design vector [6:94]. Incorrect or nonsensical designs result whenever unrealistic

constraints are placed on any system [4:3-5]. In a robot manipulator, designers must consider the structural vibration caused by the distributed link flexibility and the coupling between electromechanical drives and compliant arm linkages. Flexibility includes (1) the flexibility due to nonlinearity in the drive system and (2) flexibility due to the elasticity of the arms which have properties of a distributed parameter system. Most current robot manipulators treat the robot arms as rigid bodies driven by rigid torque generators. Such a design approach requires all components to be stiffened simultaneously, thus adding large amounts of mass to the arm. The added mass reduces all of the performance requirements previously mentioned by Kanoh [3:1866].

1.4 Optimizing Techniques

There are several different optimization techniques available to the engineer for finding an optimal solution for a given admissible set of parameters. Bandler's algorithm [1:175] optimizes several parameters by using a hybrid method. This method combines a first-order method with an approximate second-order method. The first-order method is based on linear approximation and provides good convergence in the neighborhood of the optimization solution. When a good approximation is obtained, a second-

order method can be used for a fast rate of convergence. Switching rules are given which tell when to change from the first-order to the second-order [1:175].

Komkov discusses several algorithms which design against a "worst" control [5:52-59]. He mentions the S-algorithm, the Nash approach, and the Pareto approach. The S-algorithm is not suitable for large or complex mechanical systems because of the number of computations that are required [5:52-58]. The Nash approach is inappropriate since it optimizes one parameter at the expense of all other parameters [5:59]. The Pareto approach is both cooperative and competitive [5:60].

Let us use five parameters as an example. The Pareto Method would rank each parameter in order of its importance. The first parameter is optimized and the responses of the other four parameters are checked. The parameter that is most sensitive to the first's optimization is optimized next. This procedure is continued until all five of the parameters are optimized. This method can be extended for any number of parameters that are optimized.

1.5 Safety Factor

Although solutions to optimization problems may be found, Komkov points out that we cannot always have complete

confidence in the set of data used to derive the solution. Normally a safety factor is assigned since we must deal with uncertainties such as payloads or suspect data. This factor takes care of uncertain calculations, assumptions, inaccurate modeling, and a lack of knowledge of the external loading [4:12]. These values are normally derived from years of experience. However, with modern technologies, there is no statistically significant number to draw upon for a margin of safety [4:13-15]. It is therefore impossible to determine the level of confidence in the value that is assigned as the margin of safety. In other words, the engineer's best judgement determines the margin of safety since no historical data are available.

1.6 Organization

The next chapter discusses the manipulator's design while Chapter Three discusses the Pareto optimization technique and the performance functionals that will be evaluated. We shall discuss the analysis of the functionals in Chapter Four and the Pareto Optimization results in Chapter Five. We conclude with recommendations in Chapter Six.

1.7 Summary

Optimization involves optimizing a set of parameters using one of several methods mentioned above. The Pareto method is the best method available when several parameters must be optimized concurrently. A margin of safety can be included into the solution especially when no historical data is available. The physical laws of the system must not be violated.

A parallel two degree of freedom vertically articulated manipulator was chosen for the following optimization analysis. Several performance requirements were presented for possible optimization with a discussion presented on meeting the physical constraints of the system.

II. Design of the Manipulator

Since the manipulator will be used in future telepresence research efforts by AAMRL and AFIT, we wish to emulate human arm motion. The manipulator design criteria are based on Hertzberg's results for the 50th percentile Air Force male. The final robot design will have three degrees of freedom, including one degree of freedom at the elbow and two degrees of freedom at the shoulder [9:2-1]. For our analysis, we use two degrees of freedom. This provides upper arm and forearm analysis. Motion is realized in the horizontal and vertical planes by mounting the manipulator on a moveable bracket.

This chapter addresses issues relating to emulation of a human arm by a robotic manipulator. These issues include human arm specifications, the parallel configuration of the manipulator including; manufacturing material, cross sectional area, dimensions, probable payload, choice of manipulator variables, and the validity of dynamic equations of motion for the problem.

2.1 The Human Arm Specifications

The robotic manipulator must be able to simulate the human arm's capabilities in speed, acceleration, movements (reach all positions in the given work-space), and load-carrying capability given comparable size (dimensions).

H.T.E. Hertzberg at Armstrong Aerospace Medical Research Laboratory developed the arm length and arm motions for the 50th percentile Air Force Male [2:499,545]. The results of this study are shown in Tables 2.1 and 2.2. The arm lengths, shown in Table 2.1, are used to help determine the lengths of the manipulator lengths. The arm motions, shown in Table 2.2, are used to determine the human workspace for the average human.

Table 2.1 Average Human Arm Dimensions, [2]

Arm Section	Length (inches)
Shoulder to Elbow	14.3
Elbow to Handtip	18.9
Wrist to Handtip	7.5

Table 2.2 Average Human Range of Motion, [2]

Motion	Range degrees
Shoulder Bend	249
Shoulder Twist	182
Elbow Bend	142

Acceleration and velocity for the average human were analyzed by B.A. Petrov and documented in <u>Mechanical Design of Robots</u> by Rivin [10:10]. These velocities and accelerations are shown in Table 2.3.

Table 2.3 Average Velocity and Acceleration for Human Arm [10:10]

Motion	Maximum angular speed (rad/s)	Maximum angular acceleration (rad/s2)
Shoulder Bend	7.0	70
Shoulder Twist	10.0	120
Elbow Bend	17.0	300

Shoulder bend, shoulder twist, and elbow bend, as shown in the tables on the previous page, are defined in the following manner:

Shoulder Bend: The angle which causes a vertical movement of the arm.

Shoulder Twist: The angle which causes a horizontal movement of the arm.

Elbow Bend: The angle of motion caused by the movement of the elbow.

Shoulder twist and elbow bend are the angles of interest for our research.

2.2 Design Variables

For our analysis, we considered an optimization based on three design variables. They are: (1) the payload mass (m_0), (2) the length of the link (l_2), and (3) the length of time (t_f) it takes to travel along a given trajectory a predetermined distance. Since the human arm does not lift and move the same amount of mass with every movement, it was assumed the manipulator would not always lift the same mass. In this project the mass varies from 0 kg to three kg. That

range is a realistic range for flightline, light maintenance line replaceable units (LRUs). The length of link #2 has no requirement to be of a certain length to simulate the human arm and l_2 can be used to tune the system dynamics. In this study the link length l_2 was varied from 0.05 meters to 0.26 meters and represents realistic values for the physical layout of the manipulator for possible dynamic tuning. The time t_f will vary from 0.20 to 0.44 seconds which provides realistic values for the velocity and acceleration of a human arm motion. Knowledge of these variables will allow us to find a Pareto surface.

2.3 The Parallel Configuration

The parallel manipulator configuration is shown in Figure 2.1. Link lengths are based on the average human dimensions shown in Table 2.1. The lengths for the system are:

l_1:	The length of link one	= .36322 meters
l_2:	The length of link two	variable
l_3:	The length of link three	= .36322 meters
l_4:	The length of link four	= .48006 meters

The center of gravity for all links will carry the additional subscript c. The link #2 length is one of the chosen design variables to optimize. Varying l_2 varies the performance characteristics of the manipulator and has no specified length to simulate the human arm.

There are two motors which drive the manipulator. Both are located at the grounded point in Figure 2-1. One motor allows horizontal movement of the manipulator which is measured by the angle θ_1. The other motor is located directly behind the first motor and allows vertical movement of the manipulator by moving l_2 and its displacement is measured by the angle θ_2. By combining these two movements, any area within the work-space may be reached.

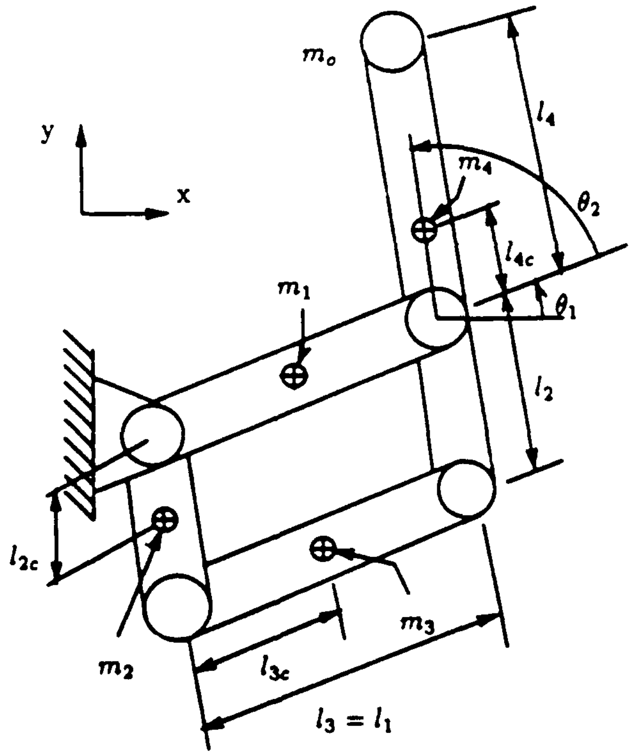

Figure 2-1 Parallel Arm Configuration [11]

In order to calculate the link's mass, the cross sectional area and the material of the manipulator must be known. The cross sections of the manipulator links were assumed to be identical rectangular channels, as shown in Figure 2-2. The human arm dimensions provide the basis for selection of this cross sectional area.

For our analysis assume that the manipulators will be constructed of aluminum which is readily available and relatively inexpensive. The density (rho) for aluminum is $2.8 * 10^3$ kg/m^3. From these values, the mass of each link may then be calculated using the following equation.

$$\text{mass} = \text{density} * \text{cross sectional area} * \text{length} \tag{2.1}$$

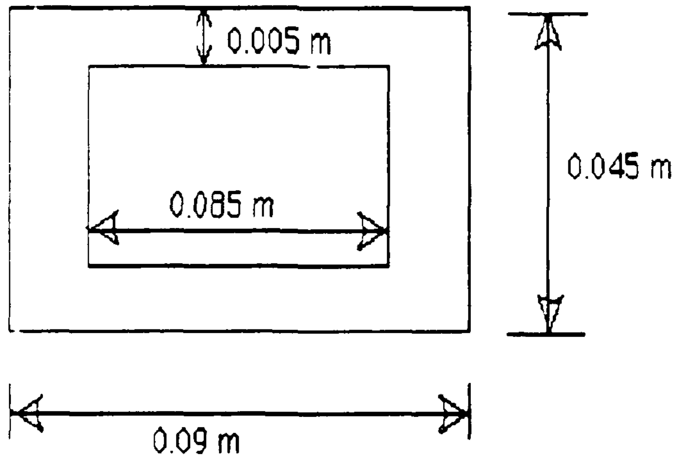

Figure 2.2 Cross Sectional Area For Each Link

The masses for the system are as follows:

m_0:	The mass of the payload		variable.
m_1:	The mass of link one	=	1.27128 kg.
m_2:	The mass of link two		variable.
m_3:	The mass of link three	=	1.27128 kg.
m_4:	The mass of link four	=	1.68021 kg.

The payload mass, m_0, is a variable since all payloads do not have the same mass. The mass m_2 is a variable since it is a function of l_2.

2.4 The Dynamic Equations of Motion

The dynamic equations of motion for a robot arm depends on the following: (1) the joint angles, (2) the joint velocities, (3) the joint accelerations, and (4) knowledge of the mechanical structure. This allows the instantaneous torques for each motor of the manipulator to be calculated.

The equations of motion for the parallel manipulator, shown in Figure 2.1, are listed below [8:2-9].

$$
\begin{aligned}
\tau_1 = [&m_1 l_{1c}^2 + I_1 + m_3 l_{3c}^2 + I_3 + m_4 l_1^2 + m_0 l_1^2 \\
&- (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_4)\cos\theta_2]\ddot{\theta}_1 \\
&- [(m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_4)\cos\theta_2]\ddot{\theta}_2 \\
&+ (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_4)\sin\theta_2 \dot{\theta}_2^2 \\
&+ (m_1 l_{1c} + m_3 l_{3c} + m_4 l_1 + m_0 l_1) g\cos\theta_1
\end{aligned}
\qquad (2.2)
$$

$$
\begin{aligned}
\tau_2 = {} & [m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_0 l_4^2 + I_0 \\
& - (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_4)\cos\theta_2]\ddot{\theta}_1 \\
& + [m_2 l_{2c}^2 + I_2 + m_3 l_2^2 + m_4 l_{4c}^2 + I_4 + m_0 l_4^2 + I_0]\ddot{\theta}_2 \\
& - (m_3 l_2 l_{3c} - m_4 l_1 l_{4c} - m_0 l_1 l_4)\sin\theta_2 \dot{\theta}_1^2 \\
& - (m_2 l_{2c} + m_3 l_2 - m_4 l_{4c} - m_0 l_4) g\cos(\theta_1+\theta_2) \qquad (2.3)
\end{aligned}
$$

where l_{1c}, l_{2c}, l_{3c}, l_{4c} are defined to be the lengths of the links to their center of gravity.

Functionals needed to find the Pareto optimal surface can be found from these equations of motion. These functionals will be discussed in detail in the next chapter.

2.5 Forward Kinematics

For our analysis, we assume only payload change at the end-effector, that is, we emulate a human holding mass in his or her hands. For a two degree of freedom manipulator the kinematic equations are as follows:

$$x = l_1 \cos(\theta_1) + l_4 \cos(\theta_1+\theta_2) \qquad (2.4)$$

$$y = l_1 \sin(\theta_1) + l_4 \sin(\theta_1+\theta_2) \qquad (2.5)$$

If the values of θ_1 and θ_2 are known, then the end-effector position in the workspace can be calculated. The manipulator's workspace is defined as the set of all x-y positions that are reachable by the manipulator. The

workspace changes only if the lengths of any of the manipulator's arms are changed.

2.6 Inverse Kinematics

Normally we know the x and y positions of the end effector rather than the joint angles (θ_1 and θ_2) but we need to know these angles for control; that is, the control input is applied at the joint level. The inverse kinematic equations, or the joint angles, that are derived from the kinematic Equations (2.4) and (2.5) are given below.

$$\theta_2 = \cos^{-1}\left[\frac{(x^2 + y^2) - (l_1^2 + l_4^2)}{2l_1l_4}\right] \tag{2.6}$$

$$\theta_1 = \tan^{-1}\left[\frac{-(l_4\sin\theta_2)x + (l_1+l_4\cos\theta_2)y}{(l_4\sin\theta_2)y + (l_1+l_4\cos\theta_2)x}\right] \tag{2.7}$$

It must be noted that the inverse tangent function is not well behaved in a full 360 degree revolution so it is sometimes necessary to add or subtract π from θ_1 depending on the x-y quadrant in which end effector lies. An additional value of π is added to the solution of Equation (2.7) when the end effector lies in the second quadrant. Conversely, if the end effector lies in the third quadrant, then π must be subtracted from the solution of Equation

(2.7). Otherwise, if the end effector lies in either the first or fourth quadrant, the solution is given in Equation (2.7).

2.7 Manipulator Trajectories

Two trajectories were analyzed for this optimization problem: (1) a vertical trajectory and (2) a horizontal trajectory. These two trajectories were chosen since their combination allows the manipulator to reach any position in its workspace. The vertical trajectory is defined as:

$$y(t) = (at^2)/2 + c_y \tag{2.8}$$

$$x(t) = c_x \tag{2.9}$$

where a is linear acceleration, t is time, c_y is the initial y position, and c_x is the initial x position. The horizontal trajectory is defined as:

$$y(t) = c_y \tag{2.10}$$

$$x(t) = (at^2)/2 + c_x \tag{2.11}$$

The inverse kinematics, Equations 2.6 and 2.7, are differentiated to find $\dot{\theta}_1$ and $\dot{\theta}_2$, the velocities, and $\ddot{\theta}_1$ and $\ddot{\theta}_2$, the accelerations. The joint velocities are:

$$\dot{\theta}_1 = \frac{l_4 \sin(\theta_1+\theta_2)\dot{y}}{l_1 l_4 \sin\theta_2} \tag{2.12}$$

$$\dot{\theta}_2 = \frac{[-l_1\sin\theta_1 - l_4\sin(\theta_1+\theta_2)]\dot{y}}{l_1 l_4 \sin\theta_2} \tag{2.13}$$

The joint accelerations are:

$$\ddot{\theta}_2 = [l_1\sin\theta_1\ddot{\theta}_1 + l_1\cos\theta_1\dot{\theta}_1^2 + l_4\sin(\theta_1+\theta_2)\ddot{\theta}_1 + l_4\cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)^2] \,/\, [-l_4\sin(\theta_1+\theta_2)] \tag{2.14}$$

$$\begin{aligned}\ddot{\theta}_1 = \{&\ddot{y} + l_4\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)^2 + [l_1\cos\theta_1\dot{\theta}_1^2 \\ &+ l_4\cos(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2)^2]\cot(\theta_1+\theta_2) \\ &+ l_1\sin\theta_1\dot{\theta}_1^2\ \} \,/\, \{l_1\cos\theta_1 + l_4\cos(\theta_1+\theta_2) \\ &- \cot(\theta_1+\theta_2)[l_1\sin\theta_1 + l_4\sin(\theta_1+\theta_2)]\}\end{aligned} \tag{2.15}$$

The functionals that will be computed use Equations (2.6) through (2.15). These functionals are discussed in detail in the next chapter.

2.8 Summary

This chapter discussed the design and specification of the manipulator. The manipulator must emulate a human arm in speed, acceleration, reach, and payload carrying capabilities. A four-bar parallel configuration of a manipulator will be used for this analysis. The physical design parameters are the payload mass, denoted m_0 and the length of the link labelled two, denoted l_2. A third design parameter is the length of time it takes to move from an initial point to a final point, denoted t_f. The dynamic

equations of motion were reviewed with equations for forward kinematics and inverse kinematics. Finally, the trajectory of the end-effector was discussed.

III. Pareto Optimality

3.1 The Pareto Minima

More than one "cost" criterion must be considered, especially in trade-off analysis, to find an optimal solution to practical optimization problems. Generally these cost criteria are $F_1(x), \ldots, F_n(x)$ and are not compatible. Then the problem becomes a vector minimization problem, that is; find:

$$\min_{x \epsilon X} F(x), \tag{3.1}$$

where

$$f_i(x) = 0 \quad \text{for all } i \epsilon N = \{1, \ldots, n\} \tag{3.2}$$

$$h_j(x) \geq 0 \quad \text{for all } j \epsilon Q = \{1, \ldots, q\} \tag{3.3}$$

and $F(x) = [F_1(x), \ldots, F_n(x)]^T$ is not a scalar but an n-dimensional vector [12:71]. Here X is a vector space.

Thus, we need to formulate a definition of a vector minima when the components are incomparable (e.g. apples and oranges). But what is meant by minimizing a vector? Obviously trying to find a minimal point where all cost components are simultaneously minimized is not only impractical but nearly impossible in most cases. For example, if a point x minimizes $F_1(x)$, it does not necessarily minimize $F_2(x)$. The method most widely used in recent times is due to Pareto (1896). The concept of

undominated or Pareto-minimal solution is essential to this theory [12:72].

Suppose we are given the previously introduced cost criteria $F_1(x),\ldots,F_n(x)$ with values $f_1(x),\ldots,f_n(x)$ at a fixed point x. We would like to know if we can move to a different point, say x^*, where the corresponding values of functionals $f_1(x),\ldots,f_n(x)$ are: $f_1(x^*),\ldots,f_n(x^*)$, such that this move would decrease the value of one or more of the functionals, while none of the other functionals would increase, that is

$$\text{for some } i, \qquad f_i(x^*) < f_i(x), \quad \text{and} \tag{3.4}$$

$$\text{for no } j,\ j=1 \text{ to } n \qquad f_j(x^*) > f_j(x) \tag{3.5}$$

If such a point x^* exists, it is definitely more desirable than the original point x. The point x^* is known as a Pareto optimal point if

(i) the physical constraints of the system are satisfied, and

(ii) no point $x=x^{**}$ exists which also satisfies the constraints such that when moving from x^* to x^{**} no function increases while at least one function decreases [7:58].

3.2 The Local Pareto Minima

There is a fundamental difference in the Pareto method and the scalar minimum concept (where a single point minimizes the function). The Pareto method eliminates the inferior points rather than seeking a single best point.

This tends to lead to continuous surfaces or hypersurfaces of local Pareto minima instead of isolated minimum points.

Definition 3.1. Let X be a normal linear space. A point $x^* \epsilon X$ is a local Pareto minimum for the vector-valued function $F(x)$ if and only if there exists a ball B centered at x^* and there does not exist a point $x \epsilon B \cap X$ with

(i) $f_j(x_*^*) < f_j(x)$ for any j, j = 1 to n, and
(ii) $f_i(x^*) > f_i(x)$ for at least one i [12:76].

3.3 Functional Analysis

Let us restate the statement of the problem: given a set of unrelated functionals, find a Pareto surface which satisfies definition 3.1. The following functionals were chosen to be optimized:

1) f_1, the Mechanical efficiency (η) or Input work (Wk_{in}) when $\eta = 0$.

2) f_2, the Peak Torque for the range of Θ_1.

3) f_3, the Peak Torque for the range of Θ_2.

3.3.1 Mechanical Efficiency

Mechanical efficiency is defined as the ratio of output work to input work.

$$\eta = \frac{|Wk_{out}|}{\sum_i |(Wk_{in})_i|} \qquad (3.6)$$

where $|Wk_{out}|$ is the absolute value of the output work and $|Wk_{in}|$ is the absolute value of the input work for each joint actuator. For our two degree of freedom manipulator, the efficiency equation may be rewritten as follows:

$$\eta = \frac{|Wk_{out}|}{|Wk_{\tau 1}| + |Wk_{\tau 2}|} \tag{3.7}$$

with $|Wk_{\tau i}|$ being the absolute value of the work supplied by the torques of each individual motor.

Mechanical efficiency may also be defined in terms of Geometric Mechanical Efficiency [11,9].

$$\eta = \frac{|Wk_{out}|}{|Wk_{out}| + \text{Geometric Work}} \tag{3.8}$$

where geometric work is the excess non-negative work done by the individual actuators to accomplish the output work. Geometric work is defined as the work consumed by all actuators minus the absolute value of the output work [11,9].

Output work is defined for a gravity lifting task in the following equation:

$$Wk_{out} = m_0 g(y_f - y_i) \tag{3.9}$$

where m_0 is the payload mass, g is gravity, y_i is the initial vertical position of the manipulator, and y_f is the final vertical position of the manipulator. If the manipulator moves from an initial position to a final position horizontally without changing vertical position, the amount of work accomplished is zero. In this special case, the mechanical efficiency functional will be of no consequence when finding the Pareto surface for the solution since in all cases it stays at its optimal value (i.e. 0).

It is necessary to find the work consumed by each actuator to find the total input work of the system as defined in Equation (3.7). The equations which calculate the work of the actuators are:

$$Wk_1 = \int \tau_1 * d\theta_1 \tag{3.10}$$

$$Wk_2 = \int \tau_2 * d\theta_2 \tag{3.11}$$

The integration limits for Equations (3.10) and (3.11) are the joint angles corresponding to the initial and final positions of the end-effector. The torques used for these equations are specified by Equations (2.2) and (2.3).

The work for each of the actuators can also be written in the following manner:

$$Wk_1 = \int (\tau_1 * \theta_1)\ dt \tag{3.12}$$

$$Wk_2 = \int (\tau_2 * \theta_2)\ dt \tag{3.13}$$

The integration limits for Equations (3.12) and (3.13) are the initial and final times of the actuator movement. The following transformation that allows this equivalence is:

$$d\theta = (d\theta/dt) * dt \tag{3.14}$$

3.3.2 Input Work

When the manipulator follows a horizontal path , the efficiency of the system is zero. Mathematically, this is obvious since $y_f - y_i = 0$ for output work. When this occurs, instead of using efficiency as the first functional, the input work of the manipulator will be considered. The input work is the amount of work consumed by the actuators. The input work for a two degree of freedom manipulator is:

$$Wk_{in} = |Wk_{\tau 1}| + |Wk_{\tau 2}| \tag{3.15}$$

The first functional, f_1, is defined in the following manner:

$$\begin{aligned} f_1 &= \eta && \text{when } y_f \neq y_i \\ &= Wk_{in} && \text{when } y_f = y_i \end{aligned} \tag{3.16}$$

3.3.3 Peak Torques

Large torques cause a long term structural weakening of the system and can cause the binding or breaking of physical components within the system. Also, larger motors would be required which could be unacceptable. Thus, one design goal is to select robot kinematics which require minimum torque for a given range of motions. The second functional is then defined as the peak value of torque one over a specified length of time. This functional is shown below:

$$f_2 = \max_{t_i \le t \le t_f} | \tau_1 | \tag{3.17}$$

where t_i is the time the manipulator starts movement and t_f is the final time period of interest. It is sufficient to check the torque in finite intervals of time such as 0.1 second increments since the change in torque is smooth.

The third functional is the peak of torque two over a specified length of time. This functional is defined below as:

$$f_3 = \max_{t_i \le t \le t_f} | \tau_2 | \tag{3.18}$$

and has t_i as initial time and t_f as final time, which corresponds to the values in the second functional. These times are equal for joint one and joint two since we specified coordinated motion.

3.4 Sensitivity Analysis

To find the Pareto optimal hypersurface, it is necessary to look at the parameter dependence of each of the functionals. The results will show which functionals are more sensitive to small changes in any of the design parameters. The matrix shown in Equation (3.19) represents the functional and design parameter combinations.

$$\begin{bmatrix} \delta f_1/\delta m_0 & \delta f_1/\delta l_2 & \delta f_1/\delta t \\ \delta f_2/\delta m_0 & \delta f_2/\delta l_2 & \delta f_2/\delta t \\ \delta f_3/\delta m_0 & \delta f_3/\delta l_2 & \delta f_3/\delta t \end{bmatrix} \tag{3.19}$$

The equations shown in the above matrix are defined below:

$$\delta f_1/\delta m_0 \approx [\eta(m_0+\delta, l_2, t) - \eta(m_0, l_2, t)] / \delta \tag{3.20}$$

$$\delta f_2/\delta m_0 \approx [f_2(m_0+\delta, l_2, t) - f_2(m_0, l_2, t)] / \delta \tag{3.21}$$

$$\delta f_3/\delta m_0 \approx [f_3(m_0+\delta, l_2, t) - f_3(m_0, l_2, t)] / \delta \tag{3.22}$$

$$\delta f_1/\delta l_2 \approx [\eta(m_0, l_2+\delta, t) - \eta(m_0, l_2, t)] / \delta \tag{3.23}$$

$$\delta f_2/\delta l_2 \approx [f_2(m_0, l_2+\delta, t) - f_2(m_0, l_2, t)] / \delta \tag{3.24}$$

$$\delta f_3/\delta l_2 \approx [f_3(m_0, l_2+\delta, t) - f_3(m_0, l_2, t)] / \delta \tag{3.25}$$

$$\delta f_1/\delta t \approx [\eta(m_0, l_2, t+\delta) - \eta(m_0, l_2, t)] / \delta \tag{3.26}$$

$$\delta f_2/\delta t \approx [f_2(m_0, l_2, t+\delta) - f_2(m_0, l_2, t)] / \delta \tag{3.27}$$

$$\delta f_3/\delta t \approx [f_3(m_0, l_2, t+\delta) - f_3(m_0, l_2, t)] / \delta \tag{3.28}$$

A change in any of the design parameters causes a change in all three functionals: $f_1(m_0,l_2,t)$, $f_2(m_0,l_2,t)$, and $f_3(m_0,l_2,t)$. The sensitivity associated with changing the payload mass, m_0, is determined by calculating Equation 3.20, Equation 3.21, and Equation 3.22. Likewise, the sensitivity of varying the length of link two, l_2, is calculated using Equation 3.23, Equation 3.24, and Equation 3.25. Equations 3.26, 3.27, and 3.28 are used to calculate the sensitivity related to a change in the length of time it takes to move from an initial point A to a final point B.

For a point to be considered a Pareto optimal point on the hypersurface, a change in one of the design parameters should cause an increase in at least one of the functionals while not allowing a decrease in any of the functionals. If the point is indeed a point on the hypersurface, then if any change to the parameter occurs, a decrease in at least one of the functionals will occur.

3.5 Summary

This chapter has addressed the Pareto optimization techniques and how to identify whether a point is located on the Pareto optimal hypersurface. The functionals to be optimized were discussed. The functionals include: mechanical efficiency, the peak torque associated with θ_1, and the peak torque associated with θ_2. If the output work is zero, then input work will be optimized rather than the

mechanical efficiency. This occurs when the manipulator moves only in the x direction. Lastly, the sensitivity analysis of the functionals was discussed.

IV. Analysis of Functionals of Pareto Optimality

Three design parameters were varied in search of a Pareto optimal surface. These parameters included: (1) the payload mass (m_0); (2) the manipulator link two length (l_2); and (3) the length of time (t_f) it takes to move the end-effector from a rest position to a position 0.1 meters away from the initial position. Three functionals were chosen to be computed when the above parameters are varied. These functionals included: (1) the mechanical efficiency of the system; (2) the peak value of the torque of the first motor; and (3) the peak value of the torque of the second motor (see Figure 2.1). The mechanical efficiency of the system is a functional while torque computations are actually point functions (they will be referred to in this document as functionals). A special case occurs when a horizontal path is followed. When this happens, the mechanical efficiency is always zero. For this trajectory, we calculate the amount of input work supplied by the system instead of mechanical efficiency for the first functional.

To perform this analysis, two trajectories are selected. The first trajectory is a vertical path of length 0.1 meters. For this path, the end-effector initially rests at the position of y = 0.45 meters and x = 0.60 meters. The end-effector is then moved vertically with the final position located at y = 0.55 meters and x = 0.60 meters.

The arm moves with constant acceleration. Thus, no deceleration occurs in reaching the final point. We computed corresponding angles, using Equations 2.6 and 2.7, for these positions. They are shown in Table 4.1. (All angles are given in radians.)

Table 4.1 Angles Corresponding to Vertical Trajectory

Angle	Initial Position (radians)	Final Position (radians)
θ_1	0.0917	0.4368
θ_2	0.9596	0.5344

The second trajectory selected is a horizontal path, which is also located in the first quadrant of the manipulator's workspace. The initial resting place for the end-effector is: y = 0.5 meters and x = 0.5 meters. The manipulator begins accelerating horizontally at a constant rate and the final position of interest is given by y = 0.5 meters and x = 0.6 meters. As above, no deceleration is assumed. The angles corresponding to the above positions are shown in Table 4.2.

Table 4.2 Angles Corresponding to Horizontal Trajectory

Angle	Initial Position (radians)	Final Position (radians)
θ_1	0.1118	0.2471
θ_2	1.1651	0.7812

For each of these trajectories, FORTRAN computer programs were written that allow a design engineer to look at computed values of a given functional for given changes in one of the design parameters. For example, an item of interest may be the sensitivity to payload mass changes when computing mechanical efficiency. The engineer would enter the values for l_2 and t_f, plus values for m_0. The program would calculate the efficiency for each payload and give the sensitivity for the payload change. These programs were written and compiled using a Samsung 300 and will execute on any IBM-PC compatible machine. In the following pages each functional and its sensitivity changes with respect to each of the design parameters are discussed. All cases which involve the vertical trajectory will be discussed first, then the behavior of all functionals will be discussed for a horizontal trajectory.

4.1 Test Cases For Each Design Parameter

As previously stated, three design parameters were varied and three functionals computed for two separate end-effector trajectories. Each design parameter used the same test case when computing the values for each functional. These test cases are given below.

4.1.1 Test Case #1: Payload Mass as the Design Parameter

The first design parameter we will vary is the mass of the payload. Sensitivity changes due to payload changes are shown in column one of the Equation 3.19 matrix. For this analysis, a constant value for t_f of 0.44 seconds is used corresponding to an acceleration of one meter per second squared. Computations of each functional are made when l_2 = 0.05, 0.10, 0.15, 0.20, and 0.25 meters. At each of these l_2 values, m_0 is varied from 0.2 to 3.2 kilograms (kg) in 0.2 kg increments.

4.1.2 Test Case #2: Link #2 Length as the Design Parameter

The design parameter to be varied for this test case is the length of link #2 of the manipulator. Sensitivity changes due to l_2 changes are shown in column two of the Equation 3.19 matrix. For this test case, a constant t_f of 0.44 seconds is again used. Computation of each functional value occurs when m_0 = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0 kilograms. At each of these payloads, the length of l_2 varies from 0.06 to 0.26 meters in 0.02 meter increments.

4.1.3 Test Case #3: t_f as the Design Parameter

The next design parameter to be varied is the time it takes the end-effector to move a distance of 0.1 meters. Sensitivity changes due to t_f changes are shown in column

three of the Equation 3.19 matrix. For this analysis, the payload mass and l_2 length are held constant and the time, t_f, it takes to move 0.1 meter varies. t_f varies from 0.20 seconds to 0.44 seconds for this analysis with an increment of 0.02 seconds. The efficiencies are evaluated for the following cases: (1) m_0 = 0.75 kgs and l_2 = 0.05 meters, (2) m_0 = 1.5 kgs and l_2 = 0.10 meters, (3) m_0 = 2.25 kgs and l_2 = 0.15 meters, and (4) m_0 = 3.00 kgs and l_2 = 0.20 meters. The acceleration is again considered constant, but since the value of t_f varies, the constant accelerations are different for each value of t_f. Table 4.3 shows the values of t_f for this portion of the analysis and the related acceleration values.

Table 4.3 Time and Associated Accelerations

Time (seconds)	Acceleration (m/s^2)
0.20	4.840
0.22	4.000
0.24	3.361
0.26	2.864
0.28	2.469
0.30	2.151
0.32	1.891
0.34	1.675
0.36	1.494
0.38	1.341
0.40	1.210
0.42	1.098
0.44	1.000

4.2 The Vertical Trajectory

4.2.1 Functional #1 - Efficiency

To compute efficiency, η, which is defined by Equation 3.6, it is necessary to know the amount of input work supplied by each of the motors. The work supplied is computed from Equations 3.12 and 3.13. We use a numerical integration technique to evaluate the input work supplied by each motor; there is no closed form solution to Equations 3.12 or 3.13. The Trapezoidal Rule is used to calculate the work values for each joint [8,142].

This rule states:

$$\int_{x1}^{xo} f(x)dx = h(.5*f_0 + f_1 + f_2 + \ldots + f_{N-1} + .5*f_N) \quad (4.1)$$

$$\text{where} \quad f_i = f(x_1+hi) \quad \text{for } i = 0,1,\ldots,N$$

The amount of torque on each joint is calculated using Equations 2.2 and 2.3. Since these torques depend on the position, velocity, and acceleration of the end-effector, it is necessary to calculate Equations 2.6, 2.7, 2.12, 2.13, 2.14, and 2.25 to get the necessary data to compute the torques.

4.2.1.1 Varying of Payload Mass

The first design parameter varied is the mass of the payload, m_0. We use test case #1 for our computations.

Computation of values for this functional show that as m_0 increases, so does the total amount of input work and efficiency. This occurs for each l_2 link. This is shown in Figure 4-1. Since efficiency increases, this means the effect of m_0 increasing outweighs the increasing value of input work, as computed from Equation 3.7. As the payload

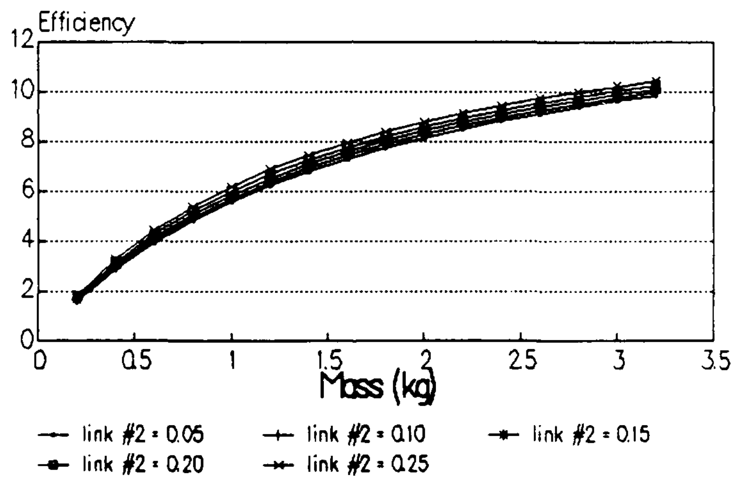

Figure 4-1 Efficiency vs Payload Mass

mass increases, the sensitivity to m_0 changes decrease, thus lighter payloads are more sensitive to changes than heavier payloads (see Figure 4-2).

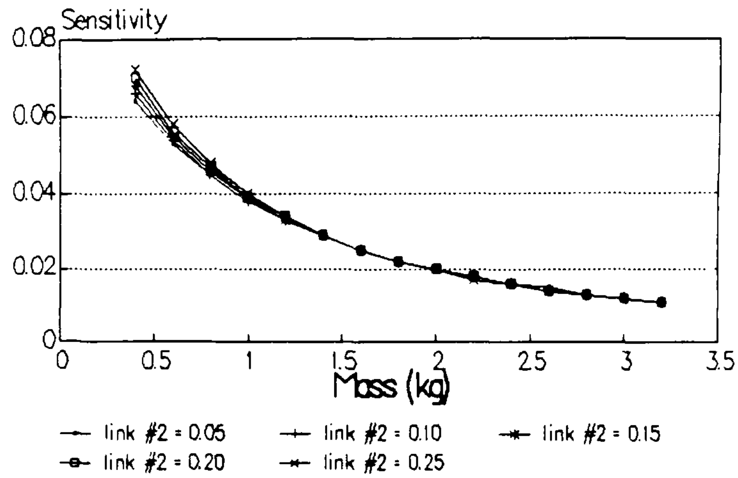

Figure 4-2 Sensitivity vs Payload Mass

4.2.1.2 Varying the Length of Link #2

The next design parameter varied is the length of the second manipulator link, l_2. We use test case #2 for these computations.

An increase in length l_2 decreases the amount of input work and slightly increases the total mechanical efficiency. This is shown in Figure 4-3. Also, l_2 is more sensitive to l_2 changes when computing efficiency as the length of l_2 is increased (see Figure 4-4).

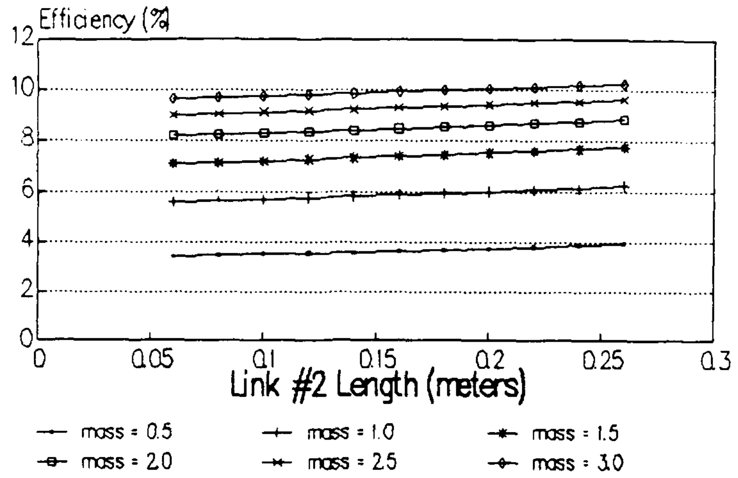

Figure 4-3 Efficiency vs Link #2 Length

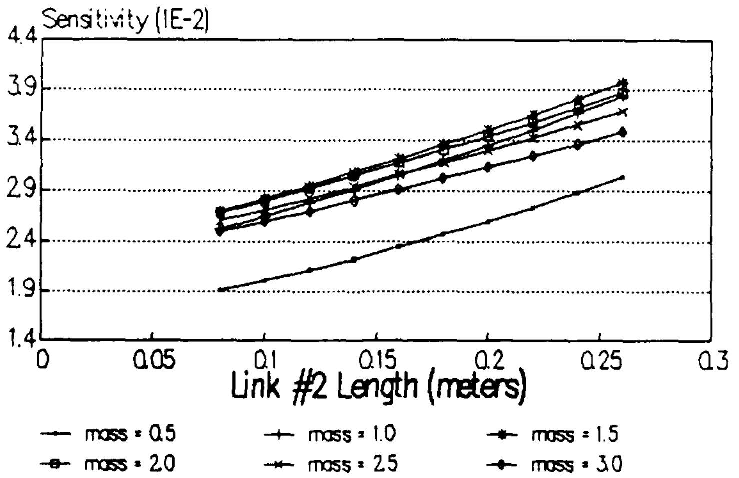

Figure 4-4 Sensitivity vs Link #2 Length

4.2.1.3 Time Variance

The next design parameter to be varied is the length of time taken to move a distance of 0.1 meters vertically. We use test case #3 for this analysis.

The values of this functional indicate that as t_f increases and acceleration decreases, the amount of input work decreases and efficiency increases. As we increase the payload mass amount and elongate link l_2, the amount of input work increases for each value of t_f. Efficiency also increases (see Figure 4-5). Thus, the values of the payload mass affects the system more than the value of the input work functional values. The system is more sensitive to

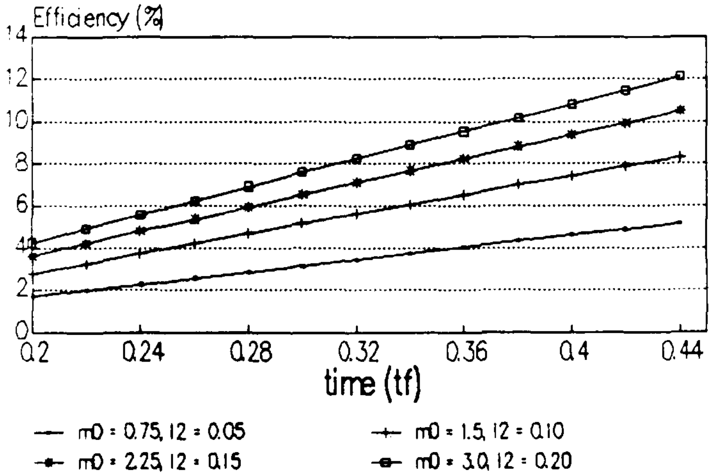

Figure 4-5 Efficiency vs t_f

changes in t_f as t_f varies from 0.20 seconds to 0.30 seconds, but as t_f becomes greater than 0.30 seconds the sensitivity decreases with an increase in the value of t_f as shown in Figure 4-6.

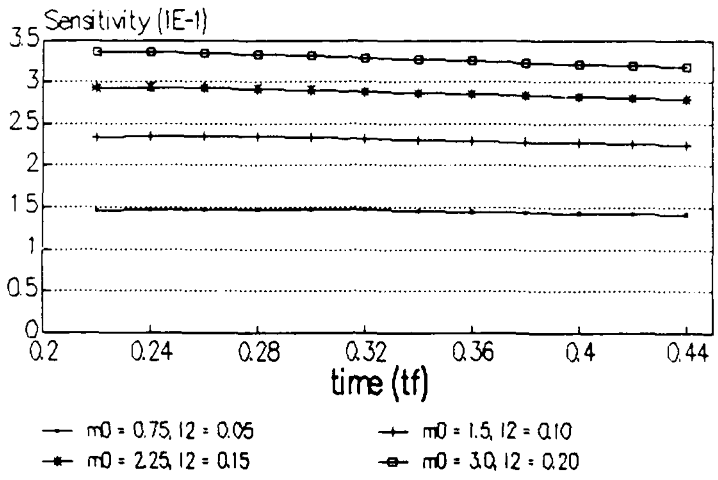

Figure 4-6 Sensitivity vs t_f

4.2.2 Functional #2 - The Maximum Value of Torque #1

This functional is defined as the peak value of torque #1 over a specified length of time (see Equation 3.17). The program computes the values of tcrque #1 from time t=0 to the final time t_f in 0.01 increments. We determined that since the torque function did not have spikes but acted smoothly, this value of time increment was sufficiently

small to accurately measure the maximum torque during the predetermined time period t_f.

4.2.2.1 Varying of Payload Mass

The design parameter varied is the payload mass. Test case #1 is used for this analysis.

The results of the functional computations show that at each specified length of l_2, the torque #1 maximum increases with an increase in payload mass (see Figure 4-7).

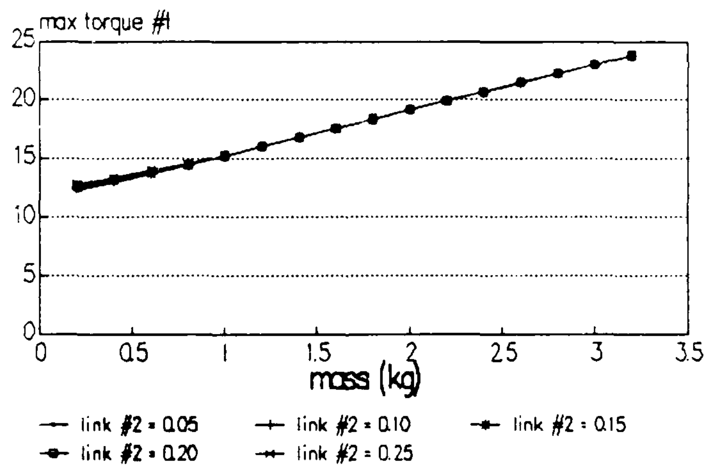

Figure 4-7 τ_1 Maximum vs Payload Mass

The sensitivity to m_0 changes was constant when payload mass is greater than 1.0 kgs for all values of length l_2.

This is shown in Figure 4-8. When the payload mass is less than 1.0 kgs, the system is not as sensitive to the 0.2 incremental changes of the payload. This leads to the conclusion that whenever m_0 is more than 1.0 kg, any incremental change in m_0 will yield the same sensitivity.

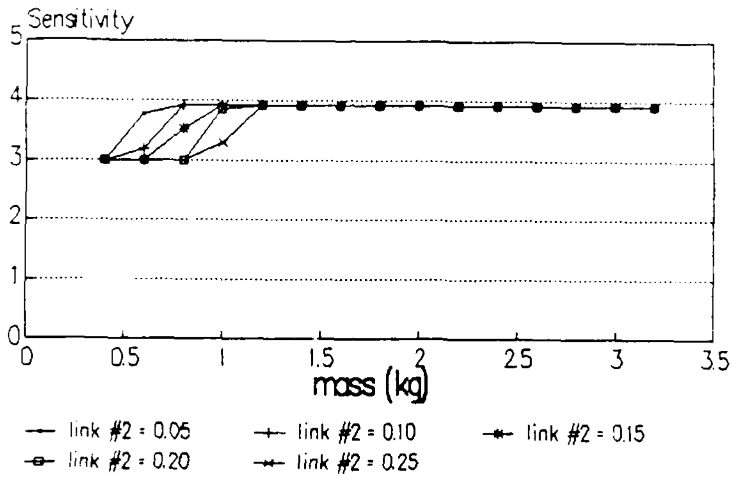

Figure 4-8 Sensitivity vs Payload Mass

4.2.2.2 Varying the Length of Link #2

The design parameter varied next for functional #2 computations is the length of manipulator link two. We use test case #2 for this analysis.

When payload mass is held constant and the length of l_2 increases in increments of 0.02 meters, a slight increase in the maximum of torque one results. This is shown in Figure 4-9.

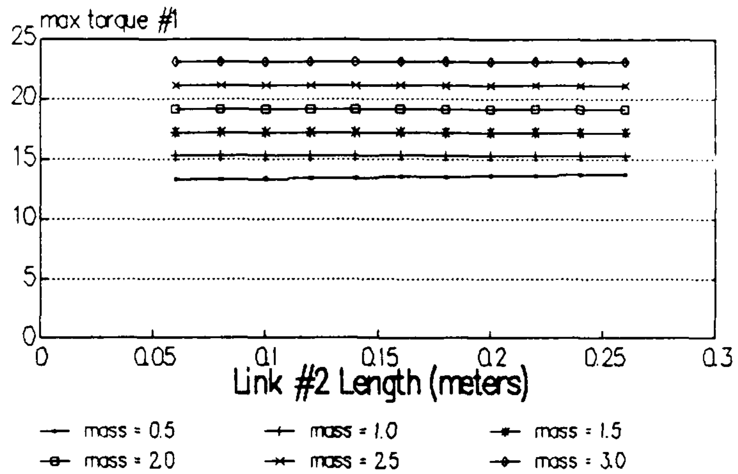

Figure 4-9 τ_1 Maximum vs Link #2 Length

For example, when the payload mass is held constant at 1.0 kgs, the maximum torque #1 value only increases by 0.00617 nt-meters. The maximum torque #1 values increase when the mass of the payload increases, but sensitivity for all payloads, except when m_0 = 0.5 kg, remains a constant value of 0.0309. When m_0 = 0.5 kg, a constant sensitivity to

changes in l_2 also occur, with the resulting sensitivity of 2.35.

4.2.2.3 Time Variance

The next design parameter varied is the length of time it takes to move the end-effector a distance of 0.1 meters vertically, denoted t_f. Our analysis uses test case #3 for this design parameter.

The results of varying t_f while holding the other design parameters constant follow. The peak value of torque #1 decreases as the manipulator has more time to move the 0.1 meters (see Figure 4-10).

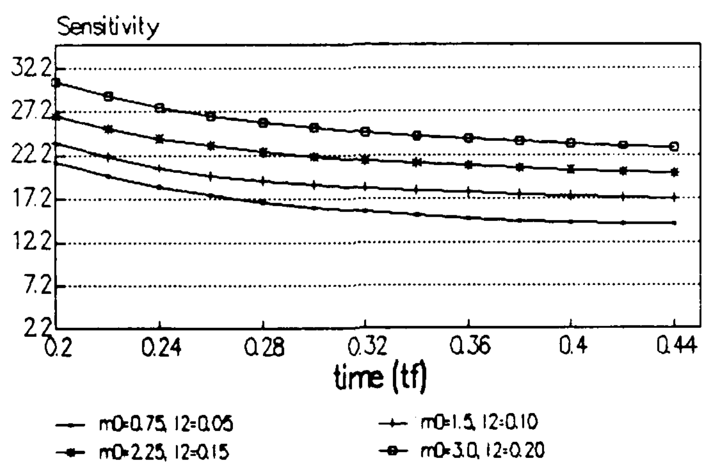

Figure 4-10 τ_1 Maximum vs t_f

The system is more sensitive to t_f changes as the trajectory time decreases. This is shown in Figure 4-11. When the length of l_2 and the payload mass are increased, the peak value of torque one also increases. The sensitivity to t_f changes are comparable for each l_2 length.

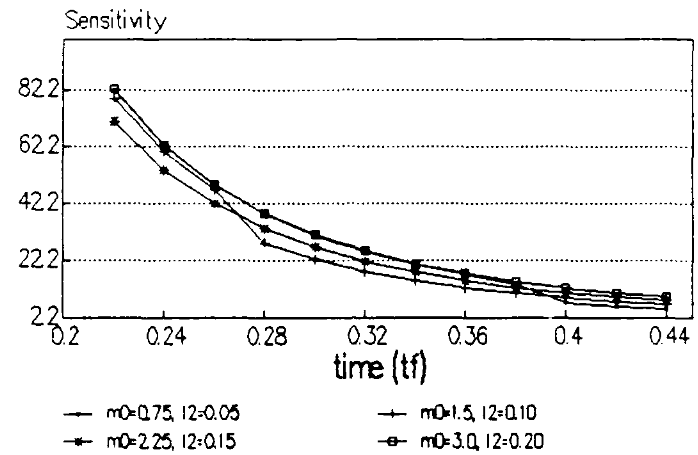

Figure 4-11 Sensitivity vs t_f

4.2.3 Functional #3 - Torque #2 Maximum

This functional is defined as the peak value of torque two over a specified length of time (see Equation 3.18). Torque #2 is evaluated at 0.01 second increments from a point when the end-effector begins movement to a time when the end-effector has moved vertically 0.1 meters. The

torques of the manipulator change values smoothly, therefore there are no times when spikes may occur during the period of time of interest. Thus, 0.01 time surveys are sufficient to find the peak torques accurately.

4.2.3.1 Varying the Payload Mass

We vary the payload mass for our analysis of the functional #3 computations in this section. Test case #1 is used for this analysis.

When the length of the second link and t_f are held constant, as payload mass increases, so does the peak value of torque two at each payload value as shown in Figure 4-12.

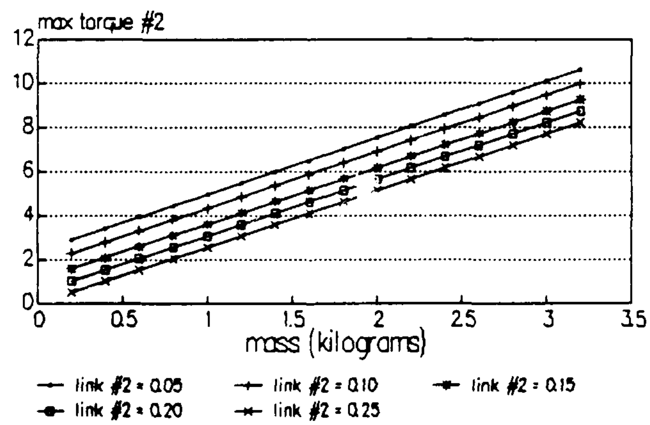

Figure 4-12 τ_2 Maximum vs Payload Mass

An increase in l_2 causes the peak value of torque #2 to decrease for each m_0 value. The changes in payload mass do not cause changes in the sensitivity. Sensitivity is a constant value of 2.57 for each link l_2 length and each payload change.

4.2.3.2 Varying the Length of Link Two

The length of link #2 of the manipulator is the design parameter we vary in this section. We use test case #2 for these computations.

As the length of link #2 increases, the peak value of torque #2 decreases at each value of m_0 evaluated. When m_0 increases in mass, the torque #2 peak values also increase. This is shown in Figure 4-13. The sensitivity to l_2 changes increase as l_2 increases from 0.06 to 0.16 meters (see Figure 4-14). The system is less sensitive to the l_2 change when l_2 goes from 0.16 to 0.18 meters. The system then continues to be more sensitive to l_2 changes as l_2 is increased above 0.18 meters.

4.2.3.3 Time Variance

The length of time (t_f) it takes for the end-effector to move from a rest position to a distance of 0.1 meters is the design parameter that will be varied for the analysis in this section. Test case #3 is used for this analysis.

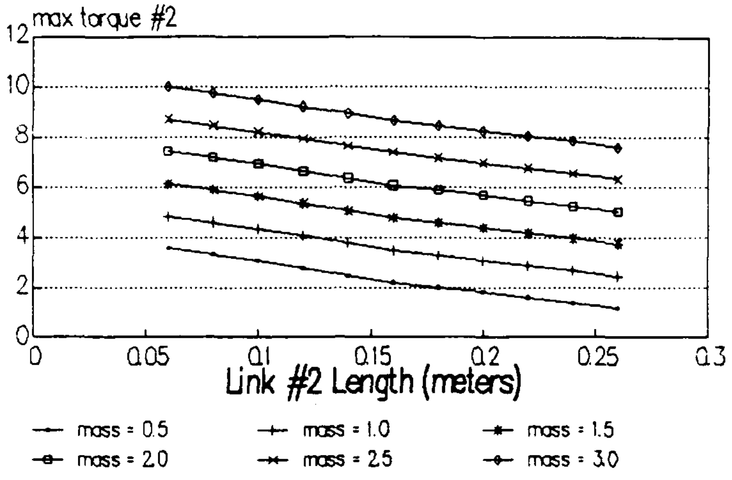

Figure 4-13 τ_2 Maximum vs Link #2 Length

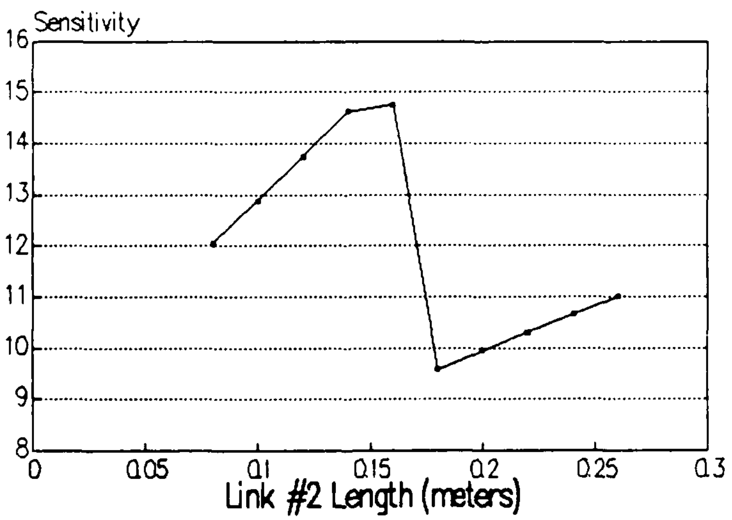

Figure 4-14 Sensitivity vs Link #2 Length

When the length of link #2 and the payload mass are held constant, the peak value of torque #2 at each t_f decreases as t_f increases. This is shown in Figure 4-15.

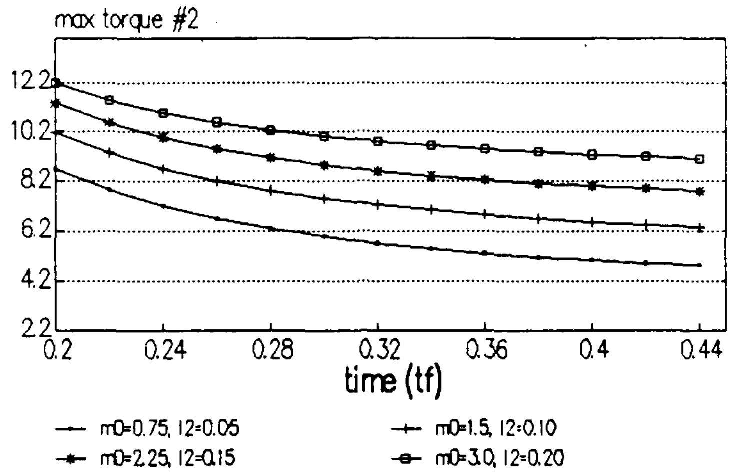

Figure 4-15 τ_2 Maximum vs t_f

The system is much more sensitive to incremental changes in t_f when less time is given the end-effector to move the necessary 0.1 meters. This is shown in Figure 4-16. As t_f increases in allowed time, torque #2 becomes less sensitive to changes in t_f. When more payload mass and a longer length for l_2 are given, the torque #2 peak value at each time increases. The sensitivity values to t_f change decrease slightly.

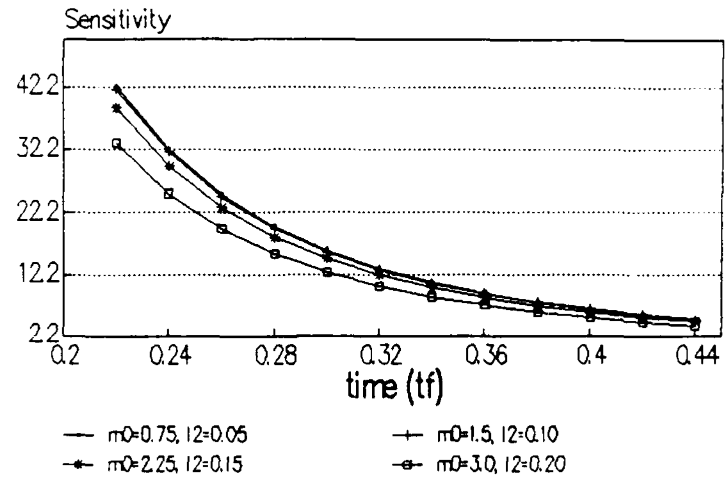

Figure 4-16 Sensitivity vs t_f

4.3 The Horizontal Trajectory

A horizontal trajectory is the second path of interest for this analysis. The end-effector is initially located at x = 0.5 meters and y = 0.5 meters in the manipulator's workspace. The end-effector is accelerated horizontally such that final position of interest occurs when x = 0.6 meters and y is still located at 0.5 meters. Constant acceleration is assumed and the joint angles corresponding to the initial and final positions of interest are located in Table 4-2.

Since the y position stays constant during this trajectory, all derivatives of y are zero. This is of particular interest since this means mathematically that a horizontal trajectory is only dependent on end-effector position, and not velocity or acceleration. This is according to equations 2.17, 2.18, 2.19, and 2.20, where $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, and $\ddot{\theta}_2$ are equal to zero since $\dot{y}$ and $\ddot{y}$ are zero.

4.3.1 Functional #1 - Input Work

In the case of the vertical trajectory, the first functional considered is system efficiency. Efficiency is defined as follows:

$$\eta = \frac{|Wk_{out}|}{\sum_i |(Wk_{in})_i|} \tag{3.6}$$

and the output work is defined as:

$$Wk_{out} = m_0 g(y_f - y_i) \tag{3.9}$$

In the case of a strictly horizontal trajectory, there is no change in y, thus $y_f - y_i = 0$ and the efficiency for this trajectory is always zero. Thus, we determined instead of efficiency being the first functional for the horizontal path, we would consider the amount of input work of each

joint angle as the first functional. Therefore, the functional of interest for this case is defined below:

$$Wk_{in} = |Wk_{\tau 1}| + |Wk_{\tau 2}| \tag{3.14}$$

where

$$Wk_1 = \int \tau_1 * d\theta_1 \tag{3.10}$$

and

$$Wk_2 = \int \tau_2 * d\theta_2 \tag{3.11}$$

Substituting into Equations 2.2 and 2.3 for the horizontal trajectory, we get the following torque equations:

$$\tau_1 = (m_1 l_{1c} + m_3 l_{3c} + m_4 l_1 + m_0 l_1) g\cos\theta_1 \tag{4.1}$$

$$\tau_2 = -(m_2 l_{2c} + m_3 l_2 + m_4 l_{4c} + m_0 l_4) g\cos(\theta_1 + \theta_2) \tag{4.2}$$

These equations can be integrated directly to find the amount of input work for each joint angle and the results are:

$$Wk_i = |(m_1 l_{1c} + m_3 l_{3c} + m_4 l_1 + m_0 l_1) g(\sin\theta_{1f} - \sin\theta_{1i})| \tag{4.3}$$

$$Wk_2 = |-(m_2 l_{2c} + m_3 l_2 + m_4 l_{4c} + m_0 l_4) g * [\sin(\theta_{1f} + \theta_{2f}) - \sin(\theta_{1i} + \theta_{2i})]| \tag{4.4}$$

These equations are used for evaluation of functional #1 with payload mass and length of link #2 the design parameters. The third design parameter is time, but this parameter is trivial when evaluating the horizontal trajectory since the horizontal path does not depend on the velocity or acceleration of the end-effector.

4.3.1.1 Varying of Payload Mass

The design parameter to vary in this section is payload mass and the first functional is now the amount of input work supplied by the manipulator instead of mechanical efficiency. Test case #1 is used for this analysis.

The results of varying the payload mass while keeping all other design parameters constant follow. When the payload mass is increased and the length of link #2 is kept constant, the amount of input work also increases (see Figure 4-17). This occurs for all values of l_2. The sensitivity of varying the payload mass is a constant value, 0.947, no matter which length of l_2 is used.

4.3.1.2 Varying the Length of Link #2

We shall next vary the length of the manipulator's link #2 while holding the other design parameters constant. Test case #2 is used for this analysis.

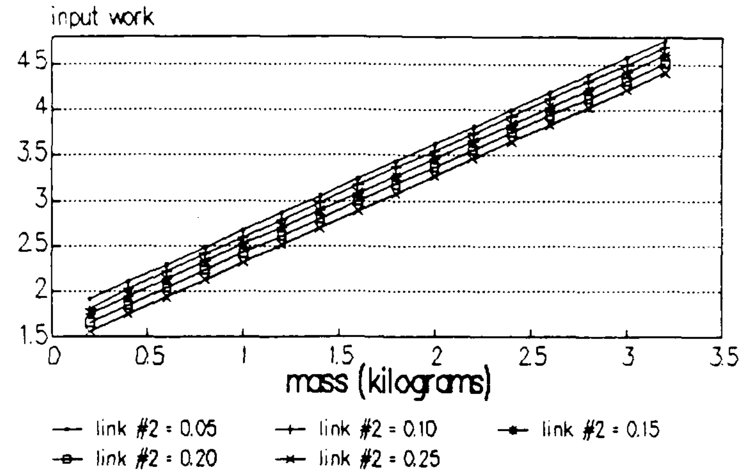

Figure 4-17 Input Work vs Payload Mass

The results of this evaluation show that when the payload mass is kept constant, the amount of input work decreases as the length of link #2 increases. This is shown in Figure 4-18. This occurs at every payload mass value. The system is more sensitive to changes in l_2 as l_2 increases in length (see Figure 4-19). When the link length changes from 0.06 to 0.08 meters, the sensitivity for this change is -1.496 for all the values of payload mass computed. This is consistent throughout the range of l_2 links shown in Figure 4-19.

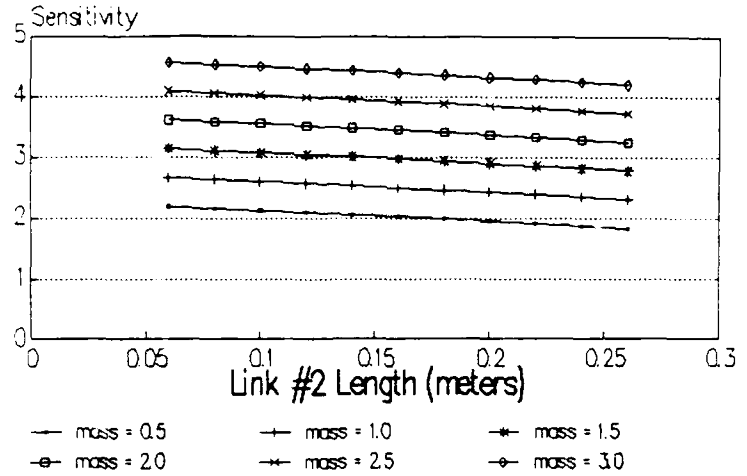

Figure 4-18 Input Work vs Link #2 Length

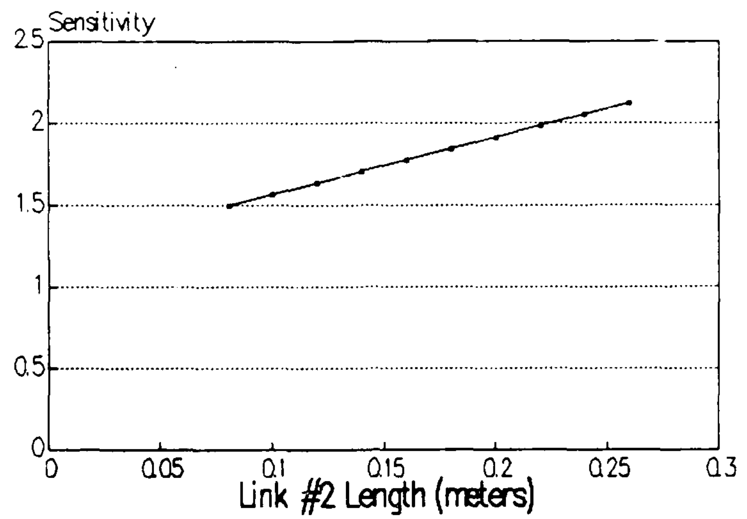

Figure 4-19 Sensitivity vs Link #2 Length

4.3.1.3 Time Variance

It was previously shown that input work does not depend on the velocity or acceleration of the manipulator, since those terms drop out of the equation when a horizontal path is used as the trajectory. That is, $\dot{y}$ and $\ddot{y}$ are zero thus $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, and $\ddot{\theta}_2$ are zero. When this occurs, equations 2.6 and 2.7 are the only non-zero inverse kinematic equations. These equations describe the position of the manipulator and are not a function of time. Therefore, the only design parameters of interest for input work computations are the payload mass and length of link #2. The functionals are not time sensitive for variance in the t_f design parameter.

4.3.2 Functional #2 - Torque #1 Maximum

This functional is defined as the peak value of torque one over a specified length of time (Equation 3.17). The program computes the values of torque one from time t=0 to $t=t_f$ in 0.01 increments of time. The torque of the joint angle is a smooth function with no spikes, thus it is sufficient to compute the torque in this interval to find the peak value within the time period of interest.

4.3.2.1 Varying of Payload Mass

Payload mass is varied while the other design parameters are held constant in this section. We use test case #1 for this analysis.

The results of varying the payload mass while holding other parameters constant show that as the payload mass increases, so does the peak value of torque #1 it takes to move the manipulator to the desired position (see Figure 4-20). It is interesting to note that for this horizontal trajectory, there is no dependence on the length of link #2. For example, the peak value for torque #1 when $m_0 = 0.2$ kgs is 11.14786 nt-meters for all lengths of l_2. The system is sensitive to changes in payload mass, but this sensitivity is constant for all mass changes. For instance, the sensitivity value for payload mass changes is 3.537 anytime m_0 is incremented by 0.2 kilograms.

4.3.2.2 Varying the Length of Link #2

The length of the manipulator's link #2 will be varied with other design parameters held constant for the evaluation occurring in this section. Test case #2 is used for this analysis.

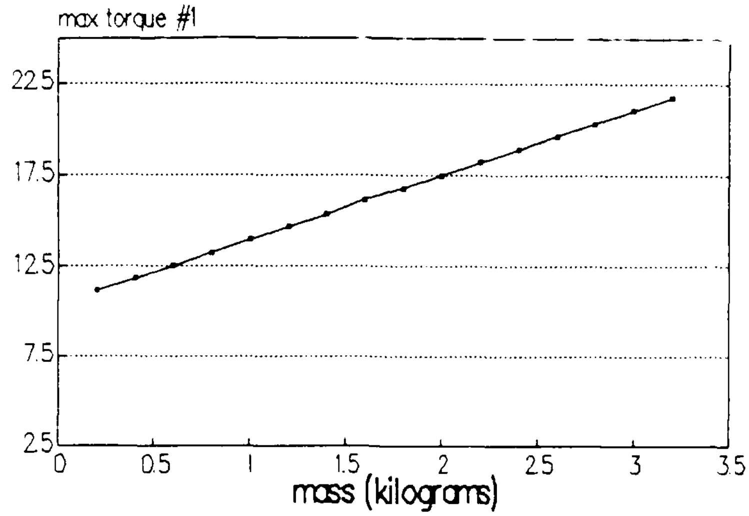

Figure 4-20 τ_1 Maximum vs Payload Mass

The analysis of the results when the length of link #2 is varied shows that for a horizontal trajectory, the system is not sensitive to changes in link l_2. That is, when other parameters are held constant and l_2 varies, the sensitivity is zero. As m_0 increases, so does the peak value of τ_1. This is shown in Table 4-4. The values for τ_1 are the same for all lengths of l_2 at each given payload.

4.3.2.3 Time Variance

For this analysis, both l_2 and m_0 are held constant, and t_f is varied. Table B-32 and Table B-33 show that the previous comments about t_f not being a factor in a

horizontal trajectory holds true. The sensitivity is zero for all changes if t_f.

Table 4-4 Payload and Resulting Maximum τ_1 Value

m_0 (kilograms)	maximum $\lvert\tau_1\rvert$ (nt-meters)
0.5	12.209
1.0	13.978
1.5	15.746
2.0	17.515
2.5	19.284
3.0	21.052

4.3.3 Functional #3 - The Maximum Value of Torque #2

The peak value of torque #2 over a specified length of time (from $t = 0$ to $t = t_f$) is the functional of interest in this section. Computations of this functional occur over the above range of time in 0.1 second increments. We determined that an accurate maximum value of torque #2 could be found with this time increment.

4.3.3.1 Varying of Payload Mass

The design parameter varied is the mass of the payload with other design parameters held constant. We use test case #1 for this analysis.

Computation of values for this functional show that as the payload mass increases, the peak torque for torque #2 also increases (see Figure 4-21). However, sensitivity is constant for each of the payload mass changes. This is also true at each l_2 length. Sensitivity equals 2.39 for all m_0 changes that occur.

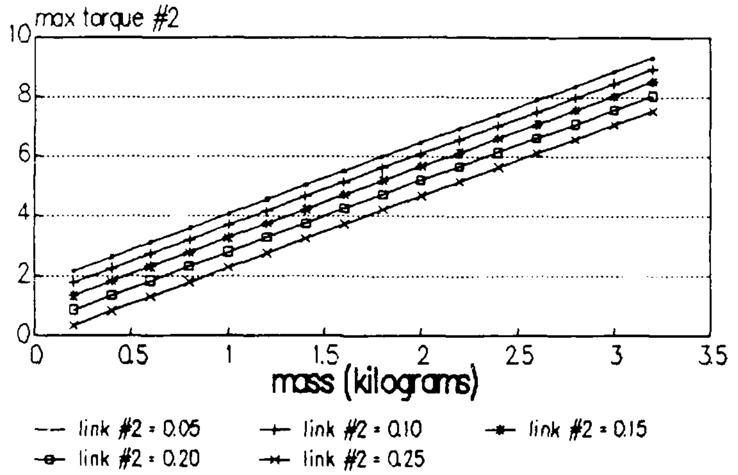

Figure 4-21 τ_2 Maximum vs Payload Mass

4.3.3.2 <u>Varying of Length of Link #2</u>

The design parameter we are interested in varying next is the length of l_2. We use test case #2 for this analysis.

As l_2 is increased in length, the corresponding maximum torque #2 value decreases (see Figure 4-22). The

sensitivity to changes in l_2's length increases as l_2 increases in length (see Figure 4-23). The sensitivity is totally independent on the value of m_0. For instance, the sensitivity is -7.5543 when l_2 increases from 0.06 to 0.08 meters for all values of m_0.

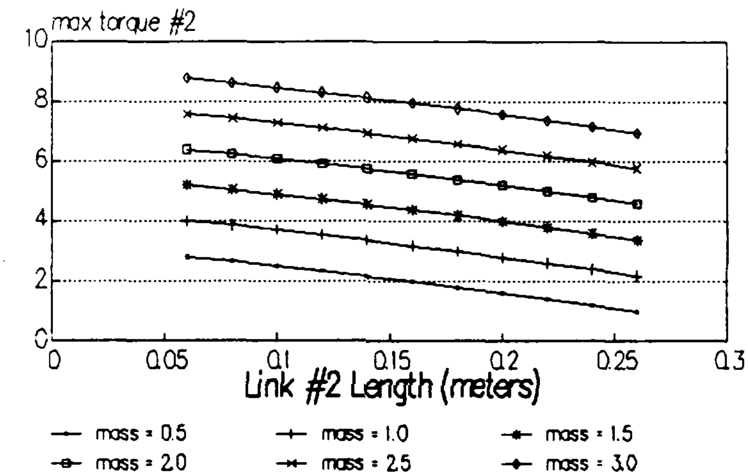

Figure 4-22 τ_2 Maximum vs Link #2 Length

4.3.5.3 <u>Time Variance</u>

The length of l_2 and the payload mass m_0 are held constant and the value of t_f is varied. As previously stated, t_f does not have any effect on the values of the functionals when the manipulator's trajectory is a horizontal path. In this case, Table B-32 and Table B-33

show a zero sensitivity when t_f changes occur with respect to the peak torque #2 values.

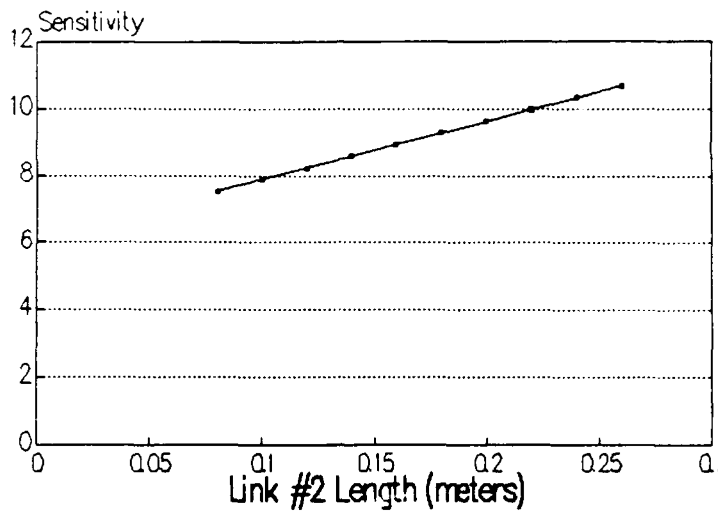

Figure 4-23 Sensitivity vs Link #2 Length

4.4 Summary

Three functionals were evaluated by varying three chosen design parameters. These functionals were: (1) mechanical efficiency of the system; (2) the peak value of the torque of the first motor; and (3) the peak value of the torque of the second motor. The design parameters that were varied are: (1) the payload mass; (2) the length of manipulator link #2; and (3) the length of time it takes for the end-effector to move 0.1 meters from a rest

position. When the manipulator moves in a horizontal path, the efficiency of the system is always zero. For this case, functional #1 is the amount of input work the motors supply rather than the efficiency of the system.

Two trajectories of the end-effector were analyzed. These trajectories included a horizontal path and a vertical path.

The vertical trajectory is more interesting than the horizontal trajectory when calculating the sensitivities to changes that occur in the design parameters. The only design parameter of interest when the horizontal path is followed is the variance of the length of manipulator link #2. The sensitivities for payload mass variance and t_f variance are either a constant value or zero for all three functionals.

However, when the vertical path is followed, several interesting results occur. Payload mass has constant sensitivity values for the torque calculations (beyond the 1.0 meter threshold), but efficiency calculations show a decrease in sensitivity when the payload increases (in 0.2 meter intervals) in mass. When l_2 is varied, computation of the peak value of torque #1 shows a constant sensitivity. The peak of torque #2 computations show the least sensitivity occurs when l_2 increases from 0.16 to 0.18

meters. Efficiency computations show an increasing sensitivity for longer l_2 lengths. As the time allowed to move the end-effector 0.1 meters increases, the system becomes less sensitive to the 0.02 second increases in t_f. The only exception to this is computation of efficiency for values of t_f less than 0.26 seconds. Chapter five will discuss the Pareto optimal conclusions which can be drawn from this analysis.

V. Optimization Results

5.1 Pareto Evaluation

If it is possible to change the value of one of the design parameters without degrading system performance with respect to other design parameters, then the parameter which is varied is not on the Pareto hypersurface. This hypersurface represents a surface in which each design parameter is at its optimal value with respect to all parameters of the system. Thus, if one of the design parameter's values is changed, a system performance degradation is expected provided the design parameters are on the optimal hypersurface.

In our analysis, we studied sensitivity changes with respect to variance of three design parameters and the resulting computations of three functionals. There were several cases when no sensitivity effects occurred with respect to other design parameters when the values of one of the parameters were varied.

Our analysis shows that when the end-effector follows a horizontal trajectory, t_f and payload mass variances do not affect the sensitivity of the system. Thus, all values for t_f and the payload mass are Pareto optimal values. This is a trivial solution since this data adds no information for finding an optimal solution.

The only design parameter (from our analysis) of interest for this trajectory, is the length of link #2. Primary interest occurs when either the peak value of torque #2 or input work values must be constrained. As the length of link #2 increases, the functionals become more sensitive to these l_2 changes. Thus, l_2 is part of the Pareto surface when the functional computations meet system constraints (stay within their required design values) and variance of l_2 does not increase system performance.

When analyzing the results of the vertical trajectory, the system is less sensitive to t_f changes as more time is given to the manipulator to move the end-effector 0.1 meters. Thus, for accuracy, we'd like to have the end-effector move slower to its destination. This design parameter, t_f, is on the Pareto surface depending on the time constraint of the system. If the system requires t_f to be short in duration, i.e. less than 0.33 seconds, small changes in the actual t_f will cause the sensitivity to those changes to be greater.

Payload mass changes are not of interest for this trajectory, since the payload may be changed and the only sensitivity changes that occur are for the efficiency computations. This does not mean that payload mass changes do not affect peak torque values. As the mass increases, so

do the values for peak torques. But these torque increases are at the same rate, which relates to a constant sensitivity value for payload changes. Thus, rather than a sensitivity constraint, the constraint for payload mass changes would be a maximum torque value.

Link #2 has a constant sensitivity for peak torque #1 computations. For functional #1 computations, as the length of link #2 increases (in 0.02 meters intervals), the sensitivity to these l_2 changes increase. Thus, when l_2 is shorter, the system is less sensitive to change than when l_2 is longer. The optimal point for a minimal sensitivity for l_2, when computing functional #3, appears to be when changing the length of l_2 from 0.16 to 0.18 meters. Other changes in l_2 yields a larger sensitivity.

5.2 Summary

Our analysis shows several cases when all design parameter values are on the Pareto surface when computing a a particular functional. For example, t_f and m_0 have all their values on the surface when the end-effector follows a horizontal trajectory. Likewise, for the vertical trajectory, payload mass changes yields a constant sensitivity computation. Thus, these design parameters are not good parameters to study for optimization of sensitivity results for the previously chosen functionals.

The length of link #2 is interesting in both the horizontal and vertical trajectory computations for sensitivity analysis. Likewise, t_f gives some interesting results.

It is important to note that the actual Pareto hypersurface for these parameters is based on all system constraints and not just the design parameters. Physical constraints, such as the manipulator staying within its workspace, as well as design constraints, such as a maximum numeric value for the torque values the designer chooses, are considered.

VI. Conclusions and Recommendations

6.1 Conclusions

A software design tool has been developed, for a parallel robotic configuration, which allows the robotics designer the ability to evaluate the sensitivities associated with changes in the following design variables: payload mass, the length of manipulator link #2, and the length of time it takes to move the end-effector a length of 0.1 meters. Three functionals were evaluated for sensitivity changes: mechanical efficiency, the peak torque of joint #1, and the peak torque of joint #2. Information concerning sensitivity is particularly valuable when the payload mass is of unknown small mass (3 kilograms or less). An analysis of the functionals with specific design parameter values was conducted with the sensitivity results shown and how the results could help to find a Pareto hypersurface. If all design parameters are on this hypersurface, then the performance of the system should be optimal for the designated design parameters. Two trajectories were analyzed: a horizontal trajectory and a vertical trajectory. When the horizontal trajectory was evaluated, input work was evaluated rather than efficiency . The results of the analysis of these two trajectories are as follows:

1. For the horizontal trajectory, changes in the design parameters payload mass and t_f have either a constant value for sensitivity or a sensitivity equal to zero for computations of all three functionals..

2. For the vertical trajectory, the peak value of torque #1 is more sensitive to changes in m_0 as m_0 increases while mechanical efficiency computations show that the system is more sensitive to changes in m_0 as m_0 decreases.

3. For the vertical trajectory, the system is more sensitive to l_2 as l_2 increases when computing mechanical efficiency. The system is least sensitive to the change when l_2 increases from 0.16 to 0.18 meters when computing the peak value of torque #2.

4. For the vertical trajectory, all functionals are less sensitive to t_f as t_f increases in time, with the exception of mechanical efficiency computations when t_f is less than 0.26 seconds.

6.2 Recommendations

The study of the following areas will allow more insight into numerical techniques available to the robotics designer. Areas of further interest include:

1. Investigation of other trajectories, including longer paths to follow for both directions.

2. Investigation of trajectories in quadrants other than the first quadrant.

3. Investigation of sensitivities for other robotic design parameters.

4. Investigation of sensitivity analysis for other robotic manipulator configurations.

Appendix A. Computer Code

A total of seven computer programs were written to generate the necessary code to do the functional analysis described in Chapter Four. Four of the programs were written for the vertical trajectory and three were written to compute values for the horizontal trajectory. The FORTRAN code is reproduced below. The code is documented for each computational case. Each program is separated by a set of '***' across the page.

*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

```
C     This Program calculates the sensitivities for functional
C     #1 when a change in either M0 or L2 occurs.
C     This program is interactive with the designer.  This is
C     the case of a vertical lift.
C
C     * * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
C     *        25 October 1989        *
C     * * * * * * * * * * * * * * * * * *
C
      REAL F1(2,100), F2(2,100), INER, T2INT(2), WKIN(2), EF(2)
      REAL L1C, L2C, L3C, L4C, L1, L2(2), L3, L4, T1INT(2)
      REAL I0, I1, I2, I3, I4
      REAL M0(2), M1, M2, M3, M4, SENSE(2)
C
C     The following steps initialize the max matrices
C
C     G is the value for gravity
C
      G = 9.8
C
C     The following values are lengths of the manipulator
C
      L1 = .36322
      L3 = .36322
      L4 = .48006
      L1C = L1/2.0
```

```
      L3C = L3/2.0
      L4C = L4/2.0
C
C     These constants allow the calculation for moments of inertia
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
      INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
C     The following values are the calculation of the masses
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Moment of inertia calculations
C
      IO = 0.0
      I1 = M1*INER
      I3 = M3*INER
      I4 = M4*INER
C
C     Choose which variable varies: M0 or L2
C
    5 WRITE (*,10)
   10 FORMAT('   Press 1 for delta T / delta M0. ')
      WRITE (*,11)
   11 FORMAT('   Press 2 for delta T / delta L2. ')
      READ (*,12) I
   12 FORMAT(I1)
      IF (I.LT.1 .OR. I.GT.2) GO TO 5
      IF (I.EQ.1 ) CALL DM0(L2(1),L2(2),M0(1),M0(2), DELTA)
      IF (I.EQ.2 ) CALL DL2(L2(1),L2(2),M0(1),M0(2), DELTA)
C
C     Choose length of time the manipulator moves
C
      WRITE (*,510)
  510 FORMAT('     Please input final time.')
      READ (*,511) TF
  511 FORMAT(F6.4)
C
C     Calculation of acceleration
C
      A = 0.1936 / (TF*TF)
C
C     This outer loop calculates the maximum torque values for the
C     two variable values.
C
C
      DO 2 I=1,2
C
      L2C = L2(I)/2.0
```

```
      M2 = L2(I)*RHO*AREA
      I2 = M2*INER
C
C     The following equations are used in calculation of the torques.
C     The A's are used for torque 1 and the B's are used for torque 2.
C
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0(I))*L1*L1
      A2 = M3*L2(I)*L3C-M4*L1*L4C-M0(I)*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0(I)*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B2 = A2
      B3 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2(I)-M4*L4C-M0(I)*L4
C
C     Initialize time
      T = 0.0
      J = 0
C
C     This loop steps through the initial time to final time to
C     find the efficiency at specific values for L2, M0.
C
C
 202  IF (T .GT. TF+.001) GO TO 201
        J = J + 1
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a vertical
C     lift of 0.1 meters with a constant x value.
C
      Y = .5*A*T*T + .45
      YD = A*T
      YDD = A
      X = .60
C
C    --------------------------------------------
C    Calculations of the angles theta1 and theta2
C    --------------------------------------------
C
      TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
      TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1(L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C    ------------------------------
C    Calculations of the velocities
C    ------------------------------
C
      TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
      TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
```

```
C
C     --------------------------
C     Values of the accelerations
C     --------------------------
C
      TS = SIN(TH1+TH2)
      TC = COS(TH1+TH2)
      TD = (TH1D+TH2D)*(TH1D+TH2D)
      TCT = 1/TAN(TH1+TH2)
C
      TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1*TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2(L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
      TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1+L4*TC*TD) / (-L4*TS)
C
C     --------------------
C     Torque 1 Calculation
C     --------------------
C
      TORQ1 = (A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1)
C
C     --------------------
C     Torque 2 Calculation
C     --------------------
C
      TORQ2 = (B1 - B2*COS(TH2)) * TH1DD + B3 * TH2DD -
     1 B4 * SIN(TH2) * TH1D * TH1D - B5 * G * COS(TH1+TH2)

      F1(I,J) = ABS(TORQ1 * TH1D)
      F2(I,J) = ABS(TORQ2 * TH2D)
C
C     Increment time interval

      T = T + .005
C
C     Loop
      GO TO 202
C
 201  CONTINUE
C
C     *** Numerical Integration ***
C
      NMI = J - 1
      DT = 1 / FLOAT(J)
C
C
      T1INT(I) = 0.5 * F1(I,1)
      Do 800 K = 2, NMI
 800  T1INT(I) = T1INT(I) + F1(I,K)
```

```
      T1INT(I) = T1INT(I) + 0.5 * F1(I,J)
      T1INT(I) = DT * T1INT(I)
C
      T2INT(I) = 0.5 * F2(I,1)
      Do 801 K = 2, NMI
 801  T2INT(I) = T2INT(I) + F2(I,K)
      T2INT(I) = T2INT(I) + 0.5 * F2(I,J)
      T2INT(I) = DT * T2INT(I)
C
      WKIN(I) = T1INT(I) + T2INT(I)
C
      EF(I) = (M0(I) * 0.10 * G) / WKIN(I)
C
   2  CONTINUE
C
C     The following gives the output.
C
      WRITE (*,111) L2(1), L2(2)
 111  FORMAT('   L2 =  ',F6.4,'      ',F6.4)
      WRITE (*,112) M0(1), M0(2)
 112  FORMAT('   M0 =  ',F6.4,'      ',F6.4)
      WRITE (*,113) TF
 113  FORMAT('  The Total Time is  ', F6.4)
C
      WRITE (*,110) A
 110  FORMAT('  The acceleration is  ', F6.3)
C
      WRITE (*,105) WKIN(1), WKIN(2)
 105  FORMAT('  Work inputs =   ', F10.5,'      ', F10.5)
      WRITE (*,106) EF(1), EF(2)
 106  FORMAT(' Efficiencies are   ', F10.5, '      ', F10.5)
C
      SENSE(1) = (EF(2) - EF(1)) / DELTA
      WRITE (*,103) SENSE(1)
 103  FORMAT('    The sensitivity  is ', F10.5)
C
C
      STOP
      END
C
C
C     This subroutine is for a varying L2.
C
      SUBROUTINE DL2(L21,L22,M01,M02,DELTA)
      REAL L21, L22, M01, M02, DELTA
      WRITE (*,401)
 401  FORMAT('     Please input initial L2.')
      READ (*,402) L21
 402  FORMAT(F6.4)
      WRITE (*,403)
 403  FORMAT('     Please input final L2.')
      READ (*,404) L22
```

```
 404   FORMAT(F6.4)
C
       WRITE (*,501)
 501   FORMAT('      Please input M0.')
       READ (*,502) M01
 502   FORMAT(F6.4)
       M02 = M01
C
       DELTA = L22 - L21
       RETURN
       END
C
C      Tnis subroutine is for a varying M0.
C
       SUBROUTINE DM0(L21,L22,M01,M02,DELTA)
       REAL L21, L22, M01, M02, DELTA
       WRITE (*,501)
 501   FORMAT('      Please input  L2.')
       READ (*,502) L21
 502   FORMAT(F6.4)
       L22 = L21
C
       WRITE (*,503)
 503   FORMAT('      Please input initial M0.')
       READ (*,504) M01
 504   FORMAT(F6.4)
C
       WRITE (*,505)
 505   FORMAT('      Please input final M0.')
       READ (*,506) M02
 506   FORMAT(F6.4)
C
       DELTA = M02 - M01
       RETURN
       END
```

*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

```
C      This program calculates the sensitivities for functional
C      One when the design parameter time is varied.
C      This program is interactive with the designer and is the
C      case of a vertical lift.
C
C      * * * * * * * * * * * * * * * * * *
C      * Author:  Jerrel D. Tumlin Jr. *
C      *      25 October 1989          *
C      * * * * * * * * * * * * * * * * * *
C
       REAL F1(2,100), F2(2,100), INER, TF(2), A(2)
       REAL L1C, L2C, L3C, L4C, L1, L2, L3, L4, T2INT(2)
```

```
      REAL I0, I1, I2, I3, I4, T1INT(2), EF(2), WKIN(2)
      REAL M0, M1, M2, M3, M4, SENSE(2)
C
C     Gravity value
C
      G = 9.8
C
C     Manipulator lengths
C
      L1 = .36322
      L1C = L1/2.0
      L3 = .36322
      L3C = L3/2.0
      L4 = .48006
      L4C = L4/2.0
C
C     Values of cross-sectional area member
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
      INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
C     Mass calculations
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Moments of Inertia calculations
C
      I0 = 0.0
      I1 = M1*INER
      I3 = M3*INER
      I4 = M4*INER
C
C     Interaction with designer for design parameters
C     These parameters include the length L2, load mass M0,
C     Final times for two separate runs.
C
      L2 = 0.0
      M0 = 0.0

      Do 901 IK = 1, 4
        L2 = L2 + 0.05
        M0 = M0 + 0.75
        TF(1) = 0.18
        TF(2) = 0.20
      DELTA = TF(2) - TF(1)
C
      L2C = L2/2.0
      M2 = L2*RHO*AREA
      I2 = M2*INER
```

```
C
C     A1 through A5 are values used to calculate torque 1.
C     B1 through B5 are values used to calculate torque 2.
C
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0)*L1*L1
      A2 = M3*L2*L3C-M4*L1*L4C-M0*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+I0
      B2 = A2
      B3 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2-M4*L4C-M0*L4
C
C     This loop allows maximum torque values to be calculated
C     for the two different final times.
C
      Do 900 IJ = 1,12
         TF(1) = TF(1) + .02
         TF(2) = TF(2) + 0.02
C
C
      DO 2 I=1,2
C
C     Acceleration calculation
C
      A(I) = 0.1936 / (TF(I)*TF(I))
C
C     Initialize time
      T = 0.00
      J = 0
C
C     A simulated while do loop to calculate max torques for a
C     specific final time.
C
 102  IF (T .GT. TF(I)+.001) GO TO 101
         J = J + 1
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a vertical
C     lift of 0.1 meters with a constant x position.
C
         Y = .5*A(I)*T*T + .45
         YD = A(I)*T
         YDD = A(I)
         X = .60
C
C     -------------------------------------
C     Values of the angles theta1 and theta2
C     -------------------------------------
```

```
C
         TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
         TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1   (L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C      ------------------------
C      Values of the velocities
C      ------------------------
C
         TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
         TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
C
C      ---------------------------
C      Values of the accelerations
C      ---------------------------
C
         TS = SIN(TH1+TH2)
         TC = COS(TH1+TH2)
         TD = (TH1D+TH2D)*(TH1D+TH2D)
         TCT = 1/TAN(TH1+TH2)
C
         TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1   *TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2   (L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
         TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1   +L4*TC*TD) / (-L4*TS)
C
C      ------------------
C      Calculate torque 1
C      ------------------
C
         TORQ1 = (A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1   A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1)
C
C      ------------------
C      Calculate torque 2
C      ------------------
C
         TORQ2 = (B1-B2*COS(TH2))*TH1DD + B3*TH2DD
     1   - B4*SIN(TH2)*TH1D*TH1D - B5*G*COS(TH1+TH2)
C
      F1(I,J) = ABS(TORQ1 * TH1D)
      F2(I,J) = ABS(TORQ2 * TH2D)
C
C      Time increment, matrix counter J
C
         T = T + 0.005
C
C      Loop
      GO TO 102
  101 CONTINUE
```

```
C
C     *** Numerical Integration ***
C
      NMI = J - 1
      DT = 1 / FLOAT(J)
C
      T1INT(I) = 0.5 * F1(I,1)
      Do 800 K = 2, NMI
 800  T1INT(I) = T1INT(I) + F1(I,K)
      T1INT(I) = T1INT(I) + 0.5 * F1(I,J)
      T1INT(I) = DT * T1INT(I)
C
      T2INT(I) = 0.5 * F2(I,1)
      Do 801 K = 2, NMI
 801  T2INT(I) = T2INT(I) + F2(I,K)
      T2INT(I) = T2INT(I) + 0.5 * F2(I,J)
      T2INT(I) = DT * T2INT(I)
C
      WKIN(I) = T1INT(I) + T2INT(I)
C
      EF(I) = (M0 * 0.10 * G) / WKIN(I)

   2  CONTINUE
C
C
C     Formatted Output
C
      WRITE (*,111) L2
 111  FORMAT('   L2 =  ',F6.4)
      WRITE (*,112) M0
 112  FORMAT('   M0 =  ',F6.4)
      WRITE (*,113) TF(1), TF(2)
 113  FORMAT('  The Total Times are  ', F6.4,'    ',F6.4)
C
      WRITE (*,110) A(1), A(2)
 110  FORMAT('  The accelerations are  ', F6.3,'    ', F6.3)
C
      WRITE (*,105) WKIN(1), WKIN(2)
 105  FORMAT('  The input work  =  ', F9.6, '      ' F9.6)
      WRITE (*,106) EF(1), EF(2)
 106  FORMAT('  The efficiencies are  ', F10.6,'    ',F10.6)

      SENSE(1) = (EF(2) - EF(1)) / DELTA
      WRITE (*,103) SENSE(1)
 103  FORMAT('    The sensitivity for efficiency is ', F10.5)
C
      WRITE (*,120)
      WRITE (*,121)
      WRITE (*,120)
 120  FORMAT('                            ')
 121  FORMAT('*****************************************')
C
```

```
C
 900  CONTINUE
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
 950  FORMAT('      ')
 901  CONTINUE
C
      STOP
      END

*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

C     This Program calculates the sensitivities for functionals
C     #2 and #3 when a change in either M0 or L2 occurs.
C     This program is interactive with the designer.  This is
C     the case of a vertical lift.
C
C     * * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
C     *       23 October 1989          *
C     * * * * * * * * * * * * * * * * * *
C
      REAL TORQ1(2), TORQ2(2), INER
      REAL L1C, L2C, L3C, L4C, L1, L2(2), L3, L4
      REAL I0, I1, I2, I3, I4
      REAL M0(2), M1, M2, M3, M4, MAX1(2), MAX2(2), SENSE(2)
C
C
C     G is the value for gravity
C
      G = 9.8
C
C     The following values are lengths of the manipulator
C
      L1 = .36322
      L3 = .36322
      L4 = .48006
      L1C = L1/2.0
      L3C = L3/2.0
      L4C = L4/2.0
C
C     These constants allow the calculation of moment of inertia
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
      INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
```

```
C     The following values are the calculation of the masses
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Moment of inertia calculations

      IO = 0.0
      I1 = M1*INER
      I3 = M3*INER
      I4 = M4*INER
C
C     Choose which variable varies: M0 or L2
C
    5 WRITE (*,10)
   10 FORMAT('   Press 1 for delta T / delta M0. ')
      WRITE (*,11)
   11 FORMAT('   Press 2 for delta T / delta L2. ')
      READ (*,12) I
   12 FORMAT(I1)
      IF (I.LT.1 .OR. I.GT.2) GO TO 5
      IF (I.EQ.1 ) CALL DM0(L2(1),L2(2),M0(1),M0(2), DELTA)
      IF (I.EQ.2 ) CALL DL2(L2(1),L2(2),M0(1),M0(2), DELTA)
C
C     Choose length of time the manipulator moves
C
      WRITE (*,510)
  510 FORMAT('     Please input final time.')
      READ (*,511) TF
  511 FORMAT(F6.4)
C
C     The following steps initialize the max matrices
C
      MAX1(1) = 0.0
      MAX1(2) = 0.0
      MAX2(1) = 0.0
      MAX2(2) = 0.0
C
C     Calculation of acceleration
C
      A = 0.1936 / (TF*TF)
C
C     This outer loop calculates the maximum torque values for the
C     two variable values.
C
      DO 2 I=1,2
C
      L2C = L2(I)/2.0
      M2 = L2(I)*RHO*AREA
      I2 = M2*INER
C
```

```
C     The following equations are used in calculation of the torques
C     The A's are used for torque 1 and the B's are used for torque 2.
C
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0(I))*L1*L1
      A2 = M3*L2(I)*L3C-M4*L1*L4C-M0(I)*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0(I)*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B2 = A2
      B3 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2(I)-M4*L4C-M0(I)*L4
C
C     Initialize time
      T=0.0
C
C     This loop steps through the initial time to final time to
C     find the maximum torques at specific values for L2, M0).
C
C
 202  IF (T .GT. TF+.001) GO TO 201
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a vertical
C     lift of 0.1 meters with a constant x value.
C
      Y = .5*A*T*T + .45
      YD = A*T
      YDD = A
      X = .60
C
C    -------------------------------------------
C    Calculations of the angles theta1 and theta2
C    -------------------------------------------
C
      TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
      TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1(L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C    -----------------------------
C    Calculations of the velocities
C    -----------------------------
C
      TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
      TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
C
C     --------------------------
C     Values of the accelerations
C     --------------------------
C
```

```
      TS = SIN(TH1+TH2)
      TC = COS(TH1+TH2)
      TD = (TH1D+TH2D)*(TH1D+TH2D)
      TCT = 1/TAN(TH1+TH2)
C
      TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1*TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2(L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
      TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1+L4*TC*TD) / (-L4*TS)
C
C     --------------------
C     Torque 1 Calculation
C     --------------------
C
      TORQ1(I) = (A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1)
C
C     Check for largest torque one value.
C
      IF (TORQ1(I) .GT. MAX1(I)) MAX1(I) = TORQ1(I)
C
C     --------------------
C     Torque 2 Calculation
C     --------------------
C
      TORQ2(I) = (B1-B2*COS(TH2))*TH1DD + B3*TH2DD -
B4*SIN(TH2)*TH1D*TH1D
     1- B5*G*COS(TH1+TH2)
C
C     Check for largest torque two value
C
      IF (TORQ2(I) .GT. MAX2(I)) MAX2(I) = TORQ2(I)
C
C     Increment time interval

      T = T + .01
C
C     Loop
      GO TO 202
C
 201  CONTINUE
   2  CONTINUE
C
C     The following gives the output.
C
      WRITE (*,111) L2(1), L2(2)
 111  FORMAT('   L2 =  ',F6.4,'      ',F6.4)
      WRITE (*,112) M0(1), M0(2)
 112  FORMAT('   M0 =  ',F6.4,'      ',F6.4)
      WRITE (*,113) TF
```

```
 113   FORMAT('  The Total Time is  ', F6.4)
C
       WRITE (*,110) A
 110   FORMAT('  The acceleration is  ', F6.3)
C
       WRITE (*,105) MAX1(1), MAX1(2)
 105   FORMAT('  Torque 1 maximums =  ', F9.6, '      ' F9.6)
       WRITE (*,106) MAX2(1), MAX2(2)
 106   FORMAT('  Torque 2 maximums =  ', F9.6, '      ', F9.6)
C
       SENSE(1) = (MAX1(2) - MAX1(1)) / DELTA
       WRITE (*,103) SENSE(1)
 103   FORMAT('    The sensitivity for torque 1 is ', F10.5)
C
       SENSE(2) = (MAX2(2) - MAX2(1)) / DELTA
       WRITE (*,104) SENSE(2)
 104   FORMAT('    The sensitivity for torque 2 is ', F10.5)
C
       STOP
       END
C
C
C      This subroutine is for a varying L2.
C
       SUBROUTINE DL2(L21,L22,M01,M02,DELTA)
       REAL L21, L22, M01, M02, DELTA
       WRITE (*,401)
 401   FORMAT('     Please input initial L2.')
       READ (*,402) L21
 402   FORMAT(F6.4)
       WRITE (*,403)
 403   FORMAT('     Please input final L2.')
       READ (*,404) L22
 404   FORMAT(F6.4)
C
       WRITE (*,501)
 501   FORMAT('     Please input M0.')
       READ (*,502) M01
 502   FORMAT(F6.4)
       M02 = M01
C
       DELTA = L22 - L21
       RETURN
       END
C
C      This subroutine is for a varying M0.
C
       SUBROUTINE DM0(L21,L22,M01,M02,DELTA)
       REAL L21, L22, M01, M02, DELTA
       WRITE (*,501)
 501   FORMAT('     Please input  L2.')
       READ (*,502) L21
```

```
 502  FORMAT(F6.4)
      L22 = L21
C
      WRITE (*,503)
 503  FORMAT('     Please input initial M0.')
      READ (*,504) M01
 504  FORMAT(F6.4)
C
      WRITE (*,505)
 505  FORMAT('     Please input final M0.')
      READ (*,506) M02
 506  FORMAT(F6.4)
C
      DELTA = M02 - M01
      RETURN
      END


*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

C     This program calculates the sensitivities for functionals
C     Two and Three when the design parameter time is varied.
C     This program is interactive with the designer and is the
C     case of a vertical lift.
C
C     * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
C     * * * * * * * * * * * * * * * * *
C
      REAL TORQ1(2), TORQ2(2), INER, TF(2), A(2)
      REAL L1C, L2C, L3C, L4C, L1, L2, L3, L4
      REAL I0, I1, I2, I3, I4
      REAL M0, M1, M2, M3, M4, MAX1(2), MAX2(2), SENSE(2)
C
C     Initialize max matrices
C
      MAX1(1) = 0.0
      MAX1(2) = 0.0
      MAX2(1) = 0.0
      MAX2(2) = 0.0
C
C     Gravity value
C
      G = 9.8
C
C     Manipulator lengths
C
      L1 = .36322
      L1C = L1/2.0
      L3 = .36322
      L3C = L3/2.0
      L4 = .48006
```

```
      L4C = L4/2.0
C
C     Values of cross-sectional area member
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
      INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
C     Mass calculations
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Moments of Inertia calculations
C
      IO = 0.0
      I1 = M1*INER
      I3 = M3*INER
      I4 = M4*INER
C
C     Interaction with designer for design parameters
C     These parameters include the length L2, load mass M0,
C     Final times for two separate runs.
C
      WRITE (*,501)
 501  FORMAT('     Please input  L2.')
      READ (*,506) L2
C
      WRITE (*,503)
 503  FORMAT('     Please input M0.')
      READ (*,506) M0
C
      WRITE (*,505)
 505  FORMAT('     Please input final time - 1.')
      READ (*,506) TF(1)
 506  FORMAT(F6.4)
C
      WRITE (*,507)
 507  FORMAT('     Please input final time - 2.')
      READ (*,508) TF(2)
 508  FORMAT(F6.4)
C
      DELTA = TF(2) - TF(1)
C
      L2C = L2/2.0
      M2 = L2*RHO*AREA
      I2 = M2*INER
C
C     A1 through A5 are values used to calculate torque 1.
C     B1 through B5 are values used to calculate torque 2.
C
```

```
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0)*L1*L1
      A2 = M3*L2*L3C-M4*L1*L4C-M0*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+I0
      B2 = A2
      B3 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2-M4*L4C-M0*L4
C
C     This loop allows maximum torque values to be calculated
C     for the two different final times.
C
      DO 2 I=1,2
C
C     Acceleration calculation
C
      A(I) = 0.1936 / (TF(I)*TF(I))
C
C     Initialize time
      T = 0.00
C
C     A simulated while do loop to calculate max torques for a
C     specific final time.
C
 102  IF (T .GT. TF(I)+.001) GO TO 101
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a vertical
C     lift of 0.1 meters with a constant x position.
C
         Y = .5*A(I)*T*T + .45
         YD = A(I)*T
         YDD = A(I)
         X = .60
C
C     --------------------------------------
C     Values of the angles theta1 and theta2
C     --------------------------------------
C
         TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
         TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1   (L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C     ----------------------
C     Values of the velocities
C     ----------------------
C
         TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
         TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
```

```
C
C     ---------------------------
C     Values of the accelerations
C     ---------------------------
C
         TS = SIN(TH1+TH2)
         TC = COS(TH1+TH2)
         TD = (TH1D+TH2D)*(TH1D+TH2D)
         TCT = 1/TAN(TH1+TH2)
C
         TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1   *TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2   (L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
         TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1   +L4*TC*TD) / (-L4*TS)
C
C     ------------------
C     Calculate torque 1
C     ------------------
C
         TORQ1(I) = ABS((A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1   A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1))
C
C     Find maximum torque one value
C
         IF (TORQ1(I) .GT. MAX1(I)) MAX1(I) = TORQ1(I)
C
C     ------------------
C     Calculate torque 2
C     ------------------
C
         TORQ2(I) = ABS((B1-B2*COS(TH2))*TH1DD + B3*TH2DD
     1   - B4*SIN(TH2)*TH1D*TH1D - B5*G*COS(TH1+TH2))
C
C     Find maximum torque two value
C
         IF (TORQ2(I) .GT. MAX2(I)) MAX2(I) = TORQ2(I)
C
C     Time increment
C
         T = T + 0.01
C
C     Loop
      GO TO 102
 101  CONTINUE
   2  CONTINUE
C
C     Formatted Output
C
      WRITE (*,111) L2
 111  FORMAT('    L2 =  ',F6.4)
```

```
      WRITE (*,112) M0
 112  FORMAT('   M0 =  ',F6.4)
      WRITE (*,113) TF(1), TF(2)
 113  FORMAT('  The Total Times are  ', F6.4,'    ',F6.4)
C
      WRITE (*,110) A(1), A(2)
 110  FORMAT('  The accelerations are  ', F6.3,'    ', F6.3)
C
      WRITE (*,105) MAX1(1), MAX1(2)
 105  FORMAT('  Torque 1 maximums =  ', F9.6, '      ' F9.6)
      WRITE (*,106) MAX2(1), MAX2(2)
 106  FORMAT('  Torque 2 maximums =  ', F9.6, '      ', F9.6)
C
      SENSE(1) = (MAX1(2) - MAX1(1)) / DELTA
      WRITE (*,103) SENSE(1)
 103  FORMAT('    The sensitivity for torque 1 is ', F10.5)
C
      SENSE(2) = (MAX2(2) - MAX2(1)) / DELTA
      WRITE (*,104) SENSE(2)
 104  FORMAT('    The sensitivity for torque 2 is ', F10.5)
C
      STOP
      END
```

*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

```
C     This Program calculates the sensitivities for functional
C     #1 when a change in either M0 or L2 occurs.
C     This program is interactive with the designer.  This is
C     the case of a horizontal push.
C
C     * * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
C     *       2 November 1989         *
C     * * * * * * * * * * * * * * * * * *
C
      REAL INER, WK1(2), WK2(2), WKIN(2)
      REAL L1C, L2C, L3C, L4C, L1, L2(2), L3, L4
      REAL I0, I1, I2, I3, I4
      REAL M0(2), M1, M2, M3, M4, SENSE(2)
C
C     G is the value for gravity
C
      G = 9.8
C
C     Initial and final angle values in radians
C
      th1f = 0.2471
      th1i = 0.1118
      th2f = 0.7812
```

```
      th2i = 1.1651
C
C     The following values are lengths of the manipulator
C
      L1 = .36322
      L3 = .36322
      L4 = .48006
      L1C = L1/2.0
      L3C = L3/2.0
      L4C = L4/2.0
C
C     These constants allow the calculation of moment of inertia
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
C
C     The following values are the calculation of the masses
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Choose which variable varies: M0 or L2
C
    5 WRITE (*,10)
   10 FORMAT('    Press 1 for delta T / delta M0. ')
      WRITE (*,11)
   11 FORMAT('    Press 2 for delta T / delta L2. ')
      READ (*,12) I
   12 FORMAT(I1)
      IF (I.LT.1 .OR. I.GT.2) GO TO 5
      IF (I.EQ.1 ) CALL DM0(L2(1),L2(2),M0(1),M0(2), DELTA)
      IF (I.EQ.2 ) CALL DL2(L2(1),L2(2),M0(1),M0(2), DELTA)
C
C     Choose length of time the manipulator moves
C
      WRITE (*,510)
  510 FORMAT('      Please input final time.')
      READ (*,511) TF
  511 FORMAT(F6.4)
C
C
C     This outer loop calculates the maximum torque values for the
C     two variable values.
C
C
             DO 2 I=1,2
C
             L2C = L2(I)/2.0
             M2 = L2(I)*RHO*AREA
             I2 = M2*INER
C
```

```
C     The following equations are used in calculation of the torques.
C     The A's are used for torque 1 and the B's are used for torque 2.
C
              A5 = M1*L1C+M3*L3C+M4*L1+M0(I)*L1
              B5 = M2*L2C+M3*L2(I)-M4*L4C-M0(I)*L4
C
C     Work calculation
C
      WK1(I) = ABS(A5*G*(sin(th1f) - sin(th1i)))
      WK2(I) = ABS(B5*G*(sin(th1f+th2f) - sin(th1i+th2i)))
C
      WKIN(I) = WK1(I) + WK2(I)
C
    2 CONTINUE
C
C
C     The following gives the output.
C
      WRITE (*,111) L2(1), L2(2)
  111 FORMAT('   L2 =  ',F6.4,'      ',F6.4)
      WRITE (*,112) M0(1), M0(2)
  112 FORMAT('   M0 =  ',F6.4,'      ',F6.4)
      WRITE (*,105) WKIN(1), WKIN(2)
  105 FORMAT(' Work inputs =   ', F10.5,'      ', F10.5)
C
      SENSE(1) = (WKIN(2) - WKIN(1)) / DELTA
      WRITE (*,103) SENSE(1)
  103 FORMAT('    The sensitivity  is ', F10.5)
      WRITE (*,120)
      WRITE (*,121)
      WRITE (*,120)
  120 FORMAT('                    ')
  121 FORMAT('****************************************')
C
C
  900 CONTINUE
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
  950 FORMAT('     ')
  901 CONTINUE
      STOP
      END
C
C     This subroutine is for a varying L2.
C
      SUBROUTINE DL2(L21,L22,M01,M02,DELTA)
      REAL L21, L22, M01, M02, DELTA
      WRITE (*,401)
  401 FORMAT('     Please input initial L2.')
```

```
      READ (*,402) L21
 402  FORMAT(F6.4)
      WRITE (*,403)
 403  FORMAT('      Please input final L2.')
      READ (*,404) L22
 404  FORMAT(F6.4)
C
      WRITE (*,501)
 501  FORMAT('      Please input M0.')
      READ (*,502) M01
 502  FORMAT(F6.4)
      M02 = M01
C
      DELTA = L22 - L21
      RETURN
      END
C
C     This subroutine is for a varying M0.
C
      SUBROUTINE DM0(L21,L22,M01,M02,DELTA)
      REAL L21, L22, M01, M02, DELTA
      WRITE (*,501)
 501  FORMAT('      Please input  L2.')
      READ (*,502) L21
 502  FORMAT(F6.4)
      L22 = L21
C
      WRITE (*,503)
 503  FORMAT('      Please input initial M0.')
      READ (*,504) M01
 504  FORMAT(F6.4)
C
      WRITE (*,505)
 505  FORMAT('      Please input final M0.')
      READ (*,506) M02
 506  FORMAT(F6.4)
C
      DELTA = M02 - M01
      RETURN
      END


*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***


C     This Program calculates the sensitivities for functionals
C     Two and Three when a change in either M0 or L2 occurs.
C     This program is interactive with the designer.  This is
C     the case of a horizontal trajectory.
C
C     * * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
```

```
C      *        23 October 1989          *
C      * * * * * * * * * * * * * * * * *
C
       REAL TORQ1(2), TORQ2(2), INER
       REAL L1C, L2C, L3C, L4C, L1, L2(2), L3, L4
       REAL I0, I1, I2, I3, I4
       REAL M0(2), M1, M2, M3, M4, MAX1(2), MAX2(2), SENSE(2)
C
C      G is the value for gravity
C
       G = 9.8
C
C      The following values are lengths of the manipulator
C
       L1 = .36322
       L3 = .36322
       L4 = .48006
       L1C = L1/2.0
       L3C = L3/2.0
       L4C = L4/2.0
C
C      These constants allow the calculation of moment of inertia
C
       RHO = 2800.0
       AREA = (.045*.09) - (.035*.08)
       INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
C      The following values are the calculation of the masses
C
       M1 = L1*RHO*AREA
       M3 = L3*RHO*AREA
       M4 = L4*RHO*AREA
C
C      Moment of inertia calculations

       IO = 0.0
       I1 = M1*INER
       I3 = M3*INER
       I4 = M4*INER
C
C      Choose which variable varies: M0 or L2
C
   5   WRITE (*,10)
  10   FORMAT('   Press 1 for delta T / delta M0. ')
       WRITE (*,11)
  11   FORMAT('   Press 2 for delta T / delta L2. ')
       READ (*,12) I
  12   FORMAT(I1)
       IF (I.LT.1 .OR. I.GT.2) GO TO 5
       IF (I.EQ.1 ) CALL DM0(L2(1),L2(2),M0(1),M0(2), DELTA)
       IF (I.EQ.2 ) CALL DL2(L2(1),L2(2),M0(1),M0(2), DELTA)
C
```

```
C     Choose length of time the manipulator moves
C
      WRITE (*,510)
 510  FORMAT('      Please input final time.')
      READ (*,511) TF
 511  FORMAT(F6.4)
C
C
C     The following steps initialize the max matrices
C
      MAX1(1) = 0.0
      MAX1(2) = 0.0
      MAX2(1) = 0.0
      MAX2(2) = 0.0
C
C     Calculation of acceleration
C
      A = 0.1936 / (TF*TF)
C
C     This outer loop calculates the maximum torque values for the
C     two variable values.
C
C
      DO 2 I=1,2
C
      L2C = L2(I)/2.0
      M2 = L2(I)*RHO*AREA
      I2 = M2*INER
C
C     The following equations are used in calculation of the torques
C     The A's are used for torque 1 and the B's are used for torque 2.
C
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0(I))*L1*L1
      A2 = M3*L2(I)*L3C-M4*L1*L4C-M0(I)*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0(I)*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B2 = A2
      B3 = M2*L2C*L2C+I2+M3*L2(I)*L2(I)+M4*L4C*L4C+I4+M0(I)*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2(I)-M4*L4C-M0(I)*L4
C
C     Initialize time
      T=0.0
C
C     This loop steps through the initial time to final time to
C     find the maximum torques at specific values for L2, M0).
C
C
 202  IF (T .GT. TF+.001) GO TO 201
```

```
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a vertical
C     lift of 0.1 meters with a constant x value.
C
      Y = 0.50
      YD = 0
      YDD = 0
      X = .5*A*T*T + .50
C
C    -------------------------------------------
C    Calculations of the angles theta1 and theta2
C    -------------------------------------------
C
      TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
      TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1(L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C    -----------------------------
C    Calculations of the velocities
C    -----------------------------
C
      TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
      TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
C
C     ---------------------------
C     Values of the accelerations
C     ---------------------------
C
      TS = SIN(TH1+TH2)
      TC = COS(TH1+TH2)
      TD = (TH1D+TH2D)*(TH1D+TH2D)
      TCT = 1/TAN(TH1+TH2)
C
      TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1*TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2(L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
      TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1+L4*TC*TD) / (-L4*TS)
C
C     -------------------
C     Torque 1 Calculation
C     -------------------
C
      TORQ1(I) = (A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1)
C
C     Check for largest torque one value.
C
      IF (TORQ1(I) .GT. MAX1(I)) MAX1(I) = TORQ1(I)
C
```

```
C     --------------------
C     Torque 2 Calculation
C     --------------------
C
      TORQ2(I) = (B1-B2*COS(TH2))*TH1DD + B3*TH2DD -
B4*SIN(TH2)*TH1D*TH1D
     1- B5*G*COS(TH1+TH2)
C
C     Check for largest torque two value
C
      IF (TORQ2(I) .GT. MAX2(I)) MAX2(I) = TORQ2(I)
C
C     Increment time interval

      T = T + .01
C
C     Loop
      GO TO 202
C
 201  CONTINUE
   2  CONTINUE
C
C     The following gives the output.
C
      WRITE (*,111) L2(1), L2(2)
 111  FORMAT('   L2 =  ',F6.4,'     ',F6.4)
      WRITE (*,112) M0(1), M0(2)
 112  FORMAT('   M0 =  ',F6.4,'     ',F6.4)
      WRITE (*,113) TF
 113  FORMAT('  The Total Time is  ', F6.4)
C
      WRITE (*,110) A
 110  FORMAT('  The acceleration is  ', F6.3)
C
      WRITE (*,105) MAX1(1), MAX1(2)
 105  FORMAT('  Torque 1 maximums =  ', F9.6, '      ' F9.6)
      WRITE (*,106) MAX2(1), MAX2(2)
 106  FORMAT('  Torque 2 maximums =  ', F9.6, '      ', F9.6)
C
      SENSE(1) = (MAX1(2) - MAX1(1)) / DELTA
      WRITE (*,103) SENSE(1)
 103  FORMAT('    The sensitivity for torque 1 is ', F10.5)
C
      SENSE(2) = (MAX2(2) - MAX2(1)) / DELTA
      WRITE (*,104) SENSE(2)
 104  FORMAT('    The sensitivity for torque 2 is ', F10.5)
C
      WRITE (*,120)
      WRITE (*,121)
      WRITE (*,120)
 120  FORMAT('                 ')
 121  FORMAT('****************************************')
```

```
C
C
 900  CONTINUE
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
      WRITE (*,950)
 950  FORMAT('     ')
 901  CONTINUE
C
      STOP
      END
C
C     This subroutine is for a varying L2.
C
      SUBROUTINE DL2(L21,L22,M01,M02,DELTA)
      REAL L21, L22, M01, M02, DELTA
      WRITE (*,401)
 401  FORMAT('     Please input initial L2.')
      READ (*,402) L21
 402  FORMAT(F6.4)
      WRITE (*,403)
 403  FORMAT('     Please input final L2.')
      READ (*,404) L22
 404  FORMAT(F6.4)
C
      WRITE (*,501)
 501  FORMAT('     Please input M0.')
      READ (*,502) M01
 502  FORMAT(F6.4)
      M02 = M01
C
      DELTA = L22 - L21
      RETURN
      END
C
C     This subroutine is for a varying M0.
C
      SUBROUTINE DM0(L21,L22,M01,M02,DELTA)
      REAL L21, L22, M01, M02, DELTA
      WRITE (*,501)
 501  FORMAT('     Please input  L2.')
      READ (*,502) L21
 502  FORMAT(F6.4)
      L22 = L21
C
      WRITE (*,503)
 503  FORMAT('     Please input initial M0.')
      READ (*,504) M01
 504  FORMAT(F6.4)
C
```

```
      WRITE (*,505)
 505  FORMAT('     Please input final MO.')
      READ (*,506) MO2
 506  FORMAT(F6.4)
C
      DELTA = MO2 - MO1
      RETURN
      END
```

*** *** *** *** *** *** *** *** *** *** *** *** *** *** *** *** ***

```
C     This program calculates the sensitivities for functionals
C     #2 and #3 when the design parameter time is varied.
C     This program is interactive with the designer and is the
C     case of a horizontal push.
C
C     * * * * * * * * * * * * * * * * *
C     * Author:  Jerrel D. Tumlin Jr. *
C     * * * * * * * * * * * * * * * * *
C
      REAL TORQ1(2), TORQ2(2), INER, TF(2), A(2)
      REAL L1C, L2C, L3C, L4C, L1, L2, L3, L4
      REAL I0, I1, I2, I3, I4
      REAL MO, M1, M2, M3, M4, MAX1(2), MAX2(2), SENSE(2)
C
C     Initialize max matrices
C
      MAX1(1) = 0.0
      MAX1(2) = 0.0
      MAX2(1) = 0.0
      MAX2(2) = 0.0
C
C     Gravity value
C
      G = 9.8
C
C     Manipulator lengths
C
      L1 = .36322
      L1C = L1/2.0
      L3 = .36322
      L3C = L3/2.0
      L4 = .48006
      L4C = L4/2.0
C
C     Values of cross-sectional area member
C
      RHO = 2800.0
      AREA = (.045*.09) - (.035*.08)
      INER = ((.09*.09 - .08*.08) - (.045*.045 - .035*.035))/12.0
C
```

```
C     Mass calculations
C
      M1 = L1*RHO*AREA
      M3 = L3*RHO*AREA
      M4 = L4*RHO*AREA
C
C     Moments of Inertia calculations
C
      IO = 0.0
      I1 = M1*INER
      I3 = M3*INER
      I4 = M4*INER
C
C     Interaction with designer for design parameters
C     These parameters include the length L2, load mass M0,
C     Final times for two separate runs.
C
      WRITE (*,501)
 501  FORMAT('     Please input  L2.')
      READ (*,506) L2
C
      WRITE (*,503)
 503  FORMAT('     Please input M0.')
      READ (*,506) M0
C
      WRITE (*,505)
 505  FORMAT('     Please input final time - 1.')
      READ (*,506) TF(1)
 506  FORMAT(F6.4)
C
      WRITE (*,507)
 507  FORMAT('     Please input final time - 2.')
      READ (*,508) TF(2)
 508  FORMAT(F6.4)
C
      DELTA = TF(2) - TF(1)
C
      L2C = L2/2.0
      M2 = L2*RHO*AREA
      I2 = M2*INER
C
C     A1 through A5 are values used to calculate torque 1.
C     B1 through B5 are values used to calculate torque 2.
C
      A1 = M1*L1C*L1C+I1+M3*L3C*L3C+I3+(M4+M0)*L1*L1
      A2 = M3*L2*L3C-M4*L1*L4C-M0*L1*L4
      A3 = A2
      A4 = A2
      A5 = M1*L1C+M3*L3C+M4*L1+M0*L1
C
      B1 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+IO
      B2 = A2
```

```
      B3 = M2*L2C*L2C+I2+M3*L2*L2+M4*L4C*L4C+I4+M0*L4*L4+I0
      B4 = B2
      B5 = M2*L2C+M3*L2-M4*L4C-M0*L4
C
C     This loop allows maximum torque values to be calculated
C     for the two different final times.
C
C
C
      DO 2 I=1,2
C
C     Acceleration calculation
C
      A(I) = 0.1936 / (TF(I)*TF(I))
C
C     Initialize time
      T = 0.00
C
C     A simulated while do loop to calculate max torques for a
C     specific final time.
C
 102  IF (T .GT. TF(I)+.001) GO TO 101
C
C     The following calculations are the X and Y positions,
C     velocities, and accelerations.  This case is a horizontal
C     push of 0.1 meters with a constant y position.
C
         Y = 0.50
         YD = 0
         YDD = 0
         X = (.5*A(I)*T*T) + 0.50
C
C     --------------------------------------
C     Values of the angles thetal and theta2
C     --------------------------------------
C
         TH2 = ACOS((X*X+Y*Y-L1*L1-L4*L4)/(2.0*L1*L4))
         TH1 = ATAN2((-L4*SIN(TH2)*X+(L1+L4*COS(TH2))*Y),
     1   (L4*SIN(TH2)*Y+(L1+L4*COS(TH2))*X))
C
C     -----------------------
C     Values of the velocities
C     -----------------------
C
         TH1D = (L4*SIN(TH1+TH2)*YD)/(L1*L4*SIN(TH2))
         TH2D = ((-L1*SIN(TH1)-L4*SIN(TH1+TH2))*YD)/(L1*L4*SIN(TH2))
C
C     --------------------------
C     Values of the accelerations
C     --------------------------
C
         TS = SIN(TH1+TH2)
```

```
          TC = COS(TH1+TH2)
          TD = (TH1D+TH2D)*(TH1D+TH2D)
          TCT = 1/TAN(TH1+TH2)
C
          TH1DD = (YDD+L4*TS*TD+(L1*COS(TH1)*TH1D*TH1D+L4*TC*TD)
     1    *TCT+L1*SIN(TH1)*TH1D*TH1D) /
     2    (L1*COS(TH1)+L4*TC-TCT*(L1*SIN(TH1)+L4*TS))
C
          TH2DD = (L1*SIN(TH1)*TH1DD+L1*COS(TH1)*TH1D*TH1D+L4*TS*TH1DD
     1    +L4*TC*TD) / (-L4*TS)
C
C     ------------------
C     Calculate torque 1
C     ------------------
C
          TORQ1(I) = ABS((A1-A2*COS(TH2))*TH1DD - A3*COS(TH2)*TH2DD +
     1    A4*SIN(TH2)*TH2D*TH2D + A5*G*COS(TH1))
C
C     Find maximum torque one value
C
          IF (TORQ1(I) .GT. MAX1(I)) MAX1(I) = TORQ1(I)
C
C     ------------------
C     Calculate torque 2
C     ------------------
C
          TORQ2(I) = ABS((B1-B2*COS(TH2))*TH1DD + B3*TH2DD
     1    - B4*SIN(TH2)*TH1D*TH1D - B5*G*COS(TH1+TH2))
C
C     Find maximum torque two value
C
          IF (TORQ2(I) .GT. MAX2(I)) MAX2(I) = TORQ2(I)
C
C     Time increment
C
          T = T + 0.01
C
C     Loop
      GO TO 102
 101  CONTINUE
   2  CONTINUE
C
C     Formatted Output
C
      WRITE (*,111) L2
 111  FORMAT('   L2 =  ',F6.4)
      WRITE (*,112) M0
 112  FORMAT('   M0 =  ',F6.4)
      WRITE (*,113) TF(1), TF(2)
 113  FORMAT(' The Total Times are  ', F6.4,'   ',F6.4)
C
      WRITE (*,110) A(1), A(2)
```

```
 110  FORMAT('  The accelerations are  ', F6.3,'    ', F6.3)
C
      WRITE (*,105) MAX1(1), MAX1(2)
 105  FORMAT('  Torque 1 maximums =  ', F9.6, '      ' F9.6)
      WRITE (*,106) MAX2(1), MAX2(2)
 106  FORMAT('  Torque 2 maximums =  ', F9.6, '      ', F9.6)
C
      SENSE(1) = (MAX1(2) - MAX1(1)) / DELTA
      WRITE (*,103) SENSE(1)
 103  FORMAT('    The sensitivity for torque 1 is ', F10.5)
C
      SENSE(2) = (MAX2(2) - MAX2(1)) / DELTA
      WRITE (*,104) SENSE(2)
 104  FORMAT('    The sensitivity for torque 2 is ', F10.5)
C
      STOP
      END
```

Appe .‿ix B. Data Tables

The following units apply for all the following tables: l_2 is in meters, t_f is in seconds, m_0 is in kilograms, work is in joules, efficiency (η) is in percentage, torques are in nt-meters, and sensitivity has no units.

l_2	t_f	m_0	$\Sigma \lvert Wk_{in} \rvert$	η	Sensitivity
0.50	0.44	0.2	12.40939	1.579	
		0.4	13.70613	2.860	0.06403
		0.6	15.00286	3.919	0.05296
		0.8	16.29960	4.810	0.04453
		1.0	17.59633	5.569	0.03797
		1.2	18.89306	6.225	0.03276
		1.4	20.18980	6.796	0.02855
		1.6	21.48653	7.298	0.02510
		1.8	22.78326	7.743	0.02225
		2.0	24.08000	8.140	0.01985
		2.2	25.37673	8.496	0.01782
		2.4	26.67347	8.818	0.01609
		2.6	27.97020	9.110	0.01460
		2.8	29.26693	9.376	0.01330
		3.0	30.56367	9.619	0.01217
		3.2	31.86040	9.843	0.01118
0.10	0.44	0.2	12.01458	1.631	
		0.4	13.31131	2.945	0.06568
		0.6	14.60805	4.025	0.05402
		0.8	15.90478	4.929	0.04521
		1.0	17.20152	5.697	0.03839
		1.2	18.49825	6.357	0.03301
		1.4	19.79498	6.931	0.02868
		1.6	21.09172	7.434	0.02516
		1.8	22.38845	7.879	0.02224
		2.0	23.68518	8.275	0.01981
		2.2	24.98192	8.630	0.01775
		2.4	26.27865	8.950	0.01600
		2.6	27.57539	9.240	0.01449
		2.8	28.87212	9.504	0.01319
		3.0	30.16885	9.745	0.01206
		3.2	31.46559	9.966	0.01106

Table B-1 Functional #1 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	$\dot{m}_0$	$\Sigma \lvert Wk_{in} \rvert$	η	Sensitivity
0.15	0.44	0.2	11.59079	1.691	
		0.4	12.88752	3.042	0.06754
		0.6	14.18426	4.145	0.05519
		0.8	15.48099	5.064	0.04594
		1.0	16.77773	5.841	0.03884
		1.2	18.07446	6.506	0.03327
		1.4	19.37119	7.083	0.02881
		1.6	20.66793	7.587	0.02520
		1.8	21.96466	8.031	0.02222
		2.0	23.26139	8.426	0.01974
		2.2	24.55813	8.779	0.01766
		2.4	25.85486	9.097	0.01589
		2.6	27.15160	9.384	0.01437
		2.8	28.44833	9.646	0.01306
		3.0	29.74507	9.884	0.01192
		3.2	31.04180	10.103	0.01093
0.20	0.44	0.2	11.13904	1.760	
		0.4	12.43578	3.152	0.06963
		0.6	13.73251	4.282	0.05648
		0.8	15.02925	5.216	0.04673
		1.0	16.32598	6.003	0.03931
		1.2	17.62271	6.673	0.03353
		1.4	18.91945	7.252	0.02893
		1.6	20.21618	7.756	0.02522
		1.8	21.51291	8.200	0.02218
		2.0	22.80965	8.593	0.01966
		2.2	24.10638	8.944	0.01754
		2.4	25.403112	9.529	0.01575
		2.6	26.69985	9.543	0.01422
		2.8	27.99659	9.801	0.01290
		3.0	29.29332	10.036	0.01176
		3.2	30.59005	10.252	0.01076

Table B-2 Functional #1 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	$\Sigma \ \lvert Wk_{in} \rvert$	η	Sensitivity
0.25	0.44	0.2	10.66036	1.839	
		0.4	11.95709	3.278	0.07199
		0.6	13.25383	4.436	0.05790
		0.8	14.55056	5.388	0.04758
		1.0	15.84730	6.184	0.03980
		1.2	17.14403	6.860	0.03378
		1.4	18.44076	7.440	0.02903
		1.6	19.73750	7.944	0.02521
		1.8	21.03423	8.386	0.02210
		2.0	22.33096	8.777	0.01954
		2.2	23.62770	9.125	0.01739
		2.4	24.92443	9.437	0.01558
		2.6	26.22117	9.717	0.01404
		2.8	27.51790	9.972	0.01272
		3.0	28.81463	10.203	0.01157
		3.2	30.11137	10.415	0.01058

Table B-3 Functional #1 with Payload Mass as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	$\Sigma \lvert Wk_{in} \rvert$	η	Sensitivity
0.5	0.44	0.06	14.27790	3.432	
		0.08	14.12114	3.470	0.01905
		0.10	13.95968	3.510	0.02007
		0.12	13.79358	3.552	0.02113
		0.14	13.62292	3.597	0.02225
		0.16	13.44774	3.644	0.02343
		0.18	13.26813	3.693	0.02466
		0.20	13.08414	3.745	0.02597
		0.22	12.89585	3.800	0.02734
		0.24	12.70330	3.857	0.02880
		0.26	12.50658	3.918	0.03034
1.0	0.44	0.06	17.51973	5.594	
		0.08	17.36297	5.644	0.02525
		0.10	17.20152	5.697	0.02649
		0.12	17.03542	5.753	0.02777
		0.14	16.86475	5.811	0.02911
		0.16	16.68958	5.872	0.03050
		0.18	16.50997	5.936	0.03194
		0.20	16.32598	6.003	0.03345
		0.22	16.13768	6.073	0.03502
		0.24	15.94514	6.146	0.03667
		0.26	15.74841	6.223	0.03839
1.5	0.44	0.06	20.76157	7.080	
		0.08	20.60481	7.134	0.02693
		0.10	20.44335	7.191	0.02817
		0.12	20.27725	7.250	0.02945
		0.14	20.10659	7.311	0.03077
		0.16	19.93142	7.375	0.03213
		0.18	19.75180	7.442	0.03353
		0.20	19.56781	7.512	0.03499
		0.22	19.37951	7.585	0.03650
		0.24	19.18697	7.661	0.03806
		0.26	18.99025	7.741	0.03968

Table B-4 Functional #1 with Link l_2 as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	$\Sigma \lvert Wk_{in} \rvert$	η	Sensitivity
2.0	0.44	0.06	24.00340	8.166	
		0.08	23.84664	8.219	0.02684
		0.10	23.68518	8.275	0.02801
		0.12	23.51909	8.334	0.02922
		0.14	23.34842	8.395	0.03046
		0.16	23.17325	8.458	0.03173
		0.18	22.99364	8.524	0.03303
		0.20	22.80965	8.593	0.03438
		0.22	22.62135	8.664	0.03576
		0.24	22.42881	8.739	0.03719
		0.26	22.23208	8.816	0.03866
2.5	0.44	0.06	27.24524	8.992	
		0.08	27.08847	9.044	0.02602
		0.10	26.92702	9.099	0.02712
		0.12	26.76092	9.155	0.02824
		0.14	26.59026	9.214	0.02938
		0.16	26.41508	9.275	0.03055
		0.18	26.23547	9.339	0.03175
		0.20	26.05148	9.404	0.03298
		0.22	25.86319	9.473	0.03423
		0.24	25.67064	9.544	0.03553
		0.26	25.47392	9.618	0.03685
3.0	0.44	0.06	30.48707	9.643	
		0.08	30.33031	9.693	0.02492
		0.10	30.16885	9.745	0.02594
		0.12	30.00276	9.799	0.02697
		0.14	29.83209	9.855	0.02803
		0.16	29.65691	9.913	0.02911
		0.18	29.47730	9.974	0.03020
		0.20	29.29332	10.036	0.03132
		0.22	29.10502	10.101	0.03247
		0.24	28.91248	10.169	0.03363
		0.26	28.71575	10.238	0.03483

Table B-5 Functional #1 with Link l_2 as Design Parameter - Vertical Trajectory

m_0	l_2	tf	$\Sigma \lvert Wk_{in} \rvert$	η	Sensitivity
0.75	0.05	0.20	43.41960	1.6928	
		0.22	37.04499	1.9841	0.14564
		0.24	32.26499	2.2780	0.14697
		0.26	28.56697	2.5729	0.14745
		0.28	25.63078	2.8676	0.14737
		0.30	23.24803	3.1616	0.14696
		0.32	21.27827	3.4542	0.14633
		0.34	19.62391	3.7454	0.14560
		0.36	18.21533	4.0351	0.14482
		0.38	17.00169	4.3231	0.14402
		0.40	15.94511	4.6096	0.14323
		0.42	15.01681	4.8945	0.14248
		0.44	14.19461	5.178	0.14175
1.50	0.10	0.20	52.35963	2.8075	
		0.22	44.88781	3.2748	0.23366
		0.24	39.25612	3.7446	0.23490
		0.26	34.87827	4.2147	0.23501
		0.28	31.38684	4.6835	0.23442
		0.30	28.54193	5.1503	0.23341
		0.32	26.18132	5.6147	0.23219
		0.34	24.19191	6.0764	0.23086
		0.36	22.49277	6.0535	0.22951
		0.38	21.02462	6.9918	0.22818
		0.40	19.74315	7.4456	0.22691
		0.42	18.61461	7.8970	0.22570
		0.44	17.61291	8.3462	0.22457

Table B-6 Functional #1 with t_f as Design Parameter - Vertical Trajectory

m_0	l_2	tf	Σ $\lvert Wk_{in} \rvert$	η	Sensitivity
2.25	0.15	0.20	61.00226	3.6146	
		0.22	52.49276	4.2006	0.29298
		0.24	46.05191	4.7881	0.29375
		0.26	41.02555	5.3747	0.29331
		0.28	37.00264	5.9590	0.29217
		0.30	33.71399	6.5403	0.29064
		0.32	30.97710	7.1182	0.28893
		0.34	28.66439	7.6925	0.28716
		0.36	26.68429	8.2633	0.28541
		0.38	24.96962	8.8307	0.28372
		0.40	23.46995	9.3950	0.28213
		0.42	22.14687	9.9563	0.28063
		0.44	20.97054	10.5148	0.27925
3.00	0.20	0.20	69.32965	4.2406	
		0.22	59.84644	4.9126	0.33598
		0.24	52.64199	5.5849	0.33616
		0.26	47.00069	6.2552	0.33517
		0.28	42.47166	6.9223	0.33352
		0.30	38.75891	7.5854	0.33154
		0.32	35.66125	8.2442	0.32945
		0.34	33.03770	8.8989	0.32734
		0.36	30.78684	9.5495	0.32531
		0.38	28.83405	10.1963	0.32337
		0.40	27.12326	10.8394	0.32156
		0.42	25.61163	11.4792	0.31988
		0.44	24.26580	12.1158	0.31833

Table B-7 Functional #1 with t_f as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\|\tau_1\|$	sensitivity
0.05	0.44	0.2	12.32777	
		0.4	12.92639	2.99311
		0.6	13.67970	3.76657
		0.8	14.46096	3.90629
		1.0	15.24222	3.90629
		1.2	16.02348	3.90628
		1.4	16.80474	3.90630
		1.6	17.58599	3.90629
		1.8	18.36725	3.90629
		2.0	19.14851	3.90630
		2.2	19.92977	3.90629
		2.4	20.71103	3.90629
		2.6	21.49229	3.90630
		2.8	22.27354	3.90629
		3.0	23.05480	3.90629
		3.2	23.83606	3.90629
0.10	0.44	0.2	12.44507	
		0.4	13.04369	2.99311
		0.6	13.68125	3.18780
		0.8	14.46251	3.90629
		1.0	15.24376	3.90629
		1.2	16.02502	3.90628
		1.4	16.80628	3.90630
		1.6	17.58754	3.90629
		1.8	18.36880	3.90629
		2.0	19.15005	3.90630
		2.2	19.93131	3.90629
		2.4	20.71257	3.90629
		2.6	21.49383	3.90630
		2.8	22.27509	3.90629
		3.0	23.05634	3.90629
		3.2	23.83760	3.90629

Table B-8 Functional #2 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\|\tau_1\|$	sensitivity
0.15	0.44	0.2	12.56236	
		0.4	13.16099	2.99311
		0.6	13.75961	2.99312
		0.8	14.46405	3.52219
		1.0	15.24531	3.90629
		1.2	16.02656	3.90628
		1.4	16.80782	3.90630
		1.6	17.58908	3.90629
		1.8	18.37034	3.90629
		2.0	19.15160	3.90630
		2.2	19.93286	3.90629
		2.4	20.71411	3.90629
		2.6	21.49537	3.90630
		2.8	22.27663	3.90629
		3.0	23.05789	3.90629
		3.2	23.83915	3.90629
0.20	0.44	0.2	12.67966	
		0.4	13.27829	2.99312
		0.6	13.87691	2.99312
		0.8	14.47553	2.99312
		1.0	15.24685	3.85659
		1.2	16.02811	3.90629
		1.4	16.80937	3.90629
		1.6	17.59062	3.90629
		1.8	18.37188	3.90629
		2.0	19.15314	3.90630
		2.2	19.93440	3.90629
		2.4	20.71566	3.90629
		2.6	21.49692	3.90630
		2.8	22.27817	3.90629
		3.0	23.05943	3.90629
		3.2	23.84069	3.90629

Table B-9 Functional #2 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\lvert\tau_1\rvert$	sensitivity
0.05	0.44	0.2	12.79696	
		0.4	13.39558	2.99312
		0.6	13.99421	2.99312
		0.8	14.59283	2.99312
		1.0	15.24839	3.27782
		1.2	16.02965	3.90629
		1.4	16.81091	3.90629
		1.6	17.59217	3.90629
		1.8	18.37342	3.90629
		2.0	19.15468	3.90630
		2.2	19.93594	3.90629
		2.4	20.71720	3.90629
		2.6	21.49846	3.90630
		2.8	22.27972	3.90629
		3.0	23.03097	3.90629
		3.2	23.84223	3.90629

Table B-10 Functional #2 with Payload Mass as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	max $\lvert\tau_1\rvert$	sensitivity
0.50	0.44	0.06	13.28938	
		0.08	13.29608	0.3349
		0.10	13.34300	2.3459
		0.12	13.38992	2.3459
		0.14	13.43684	2.3460
		0.16	13.48376	2.3459
		0.18	13.53068	2.3460
		0.20	13.57760	2.3459
		0.22	13.62452	2.3459
		0.24	13.67143	2.3459
		0.26	13.71835	2.3459
1.00	0.44	0.06	15.24253	
		0.08	15.24315	0.0308
		0.10	15.24376	0.0309
		0.12	15.24438	0.0308
		0.14	15.24500	0.0308
		0.16	15.24562	0.0308
		0.18	15.24623	0.0309
		0.20	15.24685	0.0309
		0.22	15.24747	0.0308
		0.24	15.24808	0.0308
		0.26	15.24870	0.0308
1.50	0.44	0.06	17.19567	
		0.08	17.19629	0.0309
		0.10	17.19691	0.0309
		0.12	17.19753	0.0308
		0.14	17.19814	0.0309
		0.16	17.19876	0.0309
		0.18	17.19938	0.0308
		0.20	17.20000	0.0309
		0.22	17.20061	0.0308
		0.24	17.20123	0.0309
		0.26	17.20185	0.0309

Table B-11 Functional #2 with Link l_2 as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	max $\lvert\tau_1\rvert$	sensitivity
2.0	0.44	0.06	19.14882	
		0.08	19.14944	0.0308
		0.10	19.15005	0.0309
		0.12	19.15067	0.0309
		0.14	19.15129	0.0308
		0.16	19.15191	0.0308
		0.18	19.15252	0.0308
		0.20	19.15314	0.0309
		0.22	19.15376	0.0309
		0.24	19.15437	0.0308
		0.26	19.15499	0.0309
2.5	0.44	0.06	21.10196	
		0.08	21.10258	0.0309
		0.10	21.10320	0.0308
		0.12	21.10382	0.0309
		0.14	21.10443	0.0309
		0.16	21.10505	0.0308
		0.18	21.10567	0.0309
		0.20	21.10628	0.0308
		0.22	21.10690	0.0309
		0.24	21.10752	0.0309
		0.26	21.10814	0.0308
3.0	0.44	0.06	23.05511	
		0.08	23.05573	0.3090
		0.10	23.05634	0.0309
		0.12	23.05696	0.0308
		0.14	23.05758	0.0309
		0.16	23.05819	0.0308
		0.18	23.05881	0.0309
		0.20	23.05943	0.0309
		0.22	23.06005	0.0308
		0.24	23.06066	0.0309
		0.26	23.06128	0.0309

Table B-12 Functional #2 with Link l_2 as Design Parameter - Vertical Trajectory

l_2	m_0	t_f	max $\|\tau_1\|$	sensitivity
0.05	0.75	0.20	21.49409	
		0.22	19.84907	-82.251
		0.24	18.59789	-62.559
		0.26	17.62419	-48.685
		0.28	16.85158	-38.630
		0.30	16.22827	-31.165
		0.32	15.71814	-25.506
		0.34	15.29536	-21.139
		0.36	14.94107	-17.715
		0.38	14.65591	-14.258
		0.40	14.50618	- 7.486
		0.42	14.37733	- 6.443
		0.44	14.26565	- 5.584
0.10	1.50	0.20	23.57786	
		0.22	21.99373	-79.207
		0.24	20.78887	-60.243
		0.26	19.85120	-46.883
		0.28	19.28080	-28.520
		0.30	18.82942	-22.569
		0.32	18.46000	-18.471
		0.34	18.15383	-15.308
		0.36	17.89726	-12.829
		0.38	17.68012	-10.857
		0.40	17.49473	- 9.269
		0.42	17.33519	- 7.977
		0.44	17.19691	- 6.914

Table B-13 Functional #2 with t_f as Design Parameter - Vertical Trajectory

l_2	m_0	t_f	max $\|\tau_1\|$	sensitivity
0.15	2.25	0.20	26.62168	
		0.22	25.20123	-71.023
		0.24	24.12085	-54.019
		0.26	23.28007	-42.039
		0.28	22.61293	-33.357
		0.30	22.07472	-26.911
		0.32	21.63423	-22.024
		0.34	21.26917	-18.253
		0.36	20.96324	-15.296
		0.38	20.70433	-12.945
		0.40	20.48328	-11.053
		0.42	20.29305	- 9.512
		0.44	20.12817	- 8.244
0.20	3 .00	0.20	30.60054	
		0.22	28.95092	-82.481
		0.24	27.69625	-62.733
		0.26	26.71983	-48.821
		0.28	25.94506	-38.738
		0.30	25.32002	-31.252
		0.32	24.80847	-25.578
		0.34	24.38450	-21.198
		0.36	24.02922	-17.764
		0.38	23.72854	-15.034
		0.40	23.47183	-12.836
		0.42	23.25091	-11.046
		0.44	23.05943	- 9.574

Table B-14 Functional #2 with t_f as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\|\tau_2\|$	sensitivity
0.05	0.44	0.2	2.90399	
		0.4	3.41771	2.56857
		0.6	3.93142	2.56857
		0.8	4.44514	2.56857
		1.0	4.95885	2.56857
		1.2	5.47257	2.56857
		1.4	5.98628	2.56857
		1.6	6.50000	2.56857
		1.8	7.01371	2.56857
		2.0	7.52742	2.56857
		2.2	8.04114	2.56858
		2.4	8.55486	2.56857
		2.6	9.06857	2.56857
		2.8	9.58228	2.56857
		3.0	10.09600	2.56857
		3.2	10.60971	2.56857
0.10	0.44	0.2	2.29090	
		0.4	2.80462	2.56857
		0.6	3.31833	2.56857
		0.8	3.83204	2.56857
		1.0	4.34576	2.56857
		1.2	4.85947	2.56857
		1.4	5.37319	2.56857
		1.6	5.88690	2.56857
		1.8	6.40062	2.56857
		2.0	6.91433	2.56857
		2.2	7.42805	2.56857
		2.4	7.94176	2.56857
		2.6	8.45548	2.56857
		2.8	8.96919	2.56857
		3.0	9.48291	2.56857
		3.2	9.99662	2.56858

Table B-15 Functional #3 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\|\tau_2\|$	sensitivity
0.15	0.44	0.2	1.57082	
		0.4	2.08453	2.56857
		0.6	2.59825	2.56857
		0.8	3.11196	2.56857
		1.0	3.62568	2.56857
		1.2	4.13939	2.56858
		1.4	4.65311	2.56857
		1.6	5.16682	2.56857
		1.8	5.68054	2.56857
		2.0	6.19425	2.56857
		2.2	6.70797	2.56857
		2.4	7.22168	2.56857
		2.6	7.73539	2.56857
		2.8	8.24911	2.56857
		3.0	8.76282	2.56857
		3.2	9.27654	2.56857
0.20	0.44	0.2	1.03120	
		0.4	1.54594	2.57371
		0.6	2.06069	2.57371
		0.8	2.57543	2.57371
		1.0	3.09017	2.57371
		1.2	3.60491	2.57371
		1.4	4.11966	2.57371
		1.6	4.63440	2.57371
		1.8	5.14914	2.57371
		2.0	5.66388	2.57371
		2.2	6.17862	2.57371
		2.4	6.69337	2.57371
		2.6	7.20811	2.57371
		2.8	7.72285	2.57371
		3.0	8.23759	2.57371
		3.2	8.75233	2.57371

Table B-16 Functional #3 with Payload Mass as Design Parameter - Vertical Trajectory

l_2	t_f	m_0	max $\lvert\tau_2\rvert$	sensitivity
0.25	0.44	0.2	0.50287	
		0.4	1.01762	2.57371
		0.6	1.53236	2.57371
		0.8	2.04710	2.57371
		1.0	2.56184	2.57371
		1.2	3.07658	2.57371
		1.4	3.59133	2.57371
		1.6	4.10607	2.57371
		1.8	4.62081	2.57371
		2.0	5.13555	2.57371
		2.2	5.65029	2.57375
		2.4	6.16504	2.57371
		2.6	6.67978	2.57371
		2.8	7.19452	2.57371
		3.0	7.70923	2.57371
		3.2	8.22401	2.57371

Table B-17 Functional #3 with Payload Mass as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	max $\lvert \tau_2 \rvert$	sensitivity
0.50	0.44	0.06	3.56026	
		0.08	3.31927	-12.0499
		0.10	3.06147	-12.8896
		0.12	2.78656	-13.7456
		0.14	2.49421	-14.6176
		0.16	2.19382	-15.0197
		0.18	2.00213	- 9.5841
		0.20	1.80332	- 9.9409
		0.22	1.59735	-10.2982
		0.24	1.38423	-10.6559
		0.26	1.16395	-11.0142
1.00	0.44	0.06	4.84455	
		0.08	4.60355	-12.0499
		0.10	4.34576	-12.8896
		0.12	4.07085	-13.7456
		0.14	3.77850	-14.6176
		0.16	3.48067	-14.8913
		0.18	3.28899	- 9.5841
		0.20	3.09017	- 9.9409
		0.22	2.88421	-10.2982
		0.24	2.67109	-10.6560
		0.26	2.45080	-11.0142
1.50	0.44	0.06	6.12884	
		0.08	5.88784	-12.0499
		0.10	5.63005	-12.8896
		0.12	5.35513	-13.7456
		0.14	5.06278	-14.6176
		0.16	4.76753	-14.7628
		0.18	4.57584	- 9.5841
		0.20	4.37703	- 9.9409
		0.22	4.17106	-10.2982
		0.24	3.95794	-10.6560
		0.26	3.73766	-11.0142

Table B-18 Functional #3 with Link 12 as Design Parameter - Vertical Trajectory

m_0	t_f	l_2	max $\lvert\tau_2\rvert$	sensitivity
2.00	0.44	0.06	7.41312	
		0.08	7.17212	-12.0499
		0.10	6.91433	-12.8897
		0.12	6.63942	-13.7456
		0.14	6.34707	-14.6176
		0.16	6.05438	-14.6344
		0.18	5.86270	- 9.5841
		0.20	5.66388	- 9.9409
		0.22	5.45792	-10.2982
		0.24	5.24480	-10.6560
		0.26	5.02452	-11.0142
2.50	0.44	0.06	8.69741	
		0.08	8.45641	-12.0499
		0.10	8.19862	-12.8897
		0.12	7.92371	-13.7456
		0.14	7.63136	-14.6176
		0.16	7.34124	-14.5060
		0.18	7.14956	- 9.5841
		0.20	6.95074	- 9.9409
		0.22	6.74477	-10.2982
		0.24	6.53165	-10.6559
		0.26	6.31137	-11.0142
3.00	0.44	0.06	9.98170	
		0.08	9.74070	-12.0499
		0.10	9.48290	-12.8897
		0.12	9.20799	-13.7455
		0.14	8.91564	-14.6176
		0.16	8.62809	-14.3775
		0.18	8.43641	- 9.5841
		0.20	8.23759	- 9.9409
		0.22	8.03163	-10.2982
		0.24	7.81851	-10.6560
		0.26	7.59822	-11.0142

Table B-19 Functional #3 with Link l_2 as Design Parameter - Vertical Trajectory

l_2	m_0	t_f	max $\lvert\tau_2\rvert$	sensitivity
0.05	0.75	0.20	8.705644	
		0.22	7.856912	-42.4366
		0.24	7.211382	-32.2765
		0.26	6.709007	-25.1188
		0.28	6.310388	-19.9309
		0.30	5.988800	-16.0794
		0.32	5.725606	-13.1597
		0.34	5.507477	-10.9064
		0.36	5.324681	- 9.1398
		0.38	5.169983	- 7.7349
		0.40	5.037904	- 6.6039
		0.42	4.924241	- 5.6832
		0.44	4.825722	- 4.9259
0.10	1.50	0.20	10.19789	
		0.22	9.359982	-41.8953
		0.24	8.722688	-31.8647
		0.26	8.226721	-24.7984
		0.28	7.833186	-19.6767
		0.30	7.515701	-15.8743
		0.32	7.255865	-12.9918
		0.34	7.040519	-10.7673
		0.36	6.860054	- 9.0232
		0.38	6.707329	- 7.6362
		0.40	6.576936	- 6.5197
		0.42	6.464722	- 5.6107
		0.44	6.367460	- 4.8631

Table B-20 Functional #3 with t_f as Design Parameter - Vertical Trajectory

l_2	m_0	t_f	max $\lvert\tau_2\rvert$	sensitivity
0.15	2.25	0.20	11.35665	
		0.22	10.57912	-38.8767
		0.24	9.98774	-29.5689
		0.26	9.52751	-23.0116
		0.28	9.16233	-18.2590
		0.30	8.86772	-14.7305
		0.32	8.62661	-12.0558
		0.34	8.42676	- 9.9915
		0.36	8.25931	- 8.3731
		0.38	8.11759	- 7.0860
		0.40	7.99659	- 6.0499
		0.42	7.89246	- 5.2064
		0.44	7.80221	- 4.5127
0.20	3.00	0.20	12.15751	
		0.22	11.49414	-33.1689
		0.24	10.98958	-25.2277
		0.26	10.5969	-19.6332
		0.28	10.28535	-15.5782
		0.30	10.03400	-12.5677
		0.32	9.82828	-10.2858
		0.34	9.65779	- 8.5246
		0.36	9.51492	- 7.1438
		0.38	9.39400	- 6.0457
		0.40	9.29077	- 5.1617
		0.42	9.20193	- 4.4420
		0.44	9.12493	- 3.8502

Table B-21 Functional #3 with t_f as Design Parameter - Vertical Trajectory

l_2	m_0	$\Sigma \lvert Wk_{in} \rvert$	Sensitivity
0.05	0.20	1.91797	
	0.40	2.10743	0.94727
	0.60	2.29688	0.94726
	0.80	2.48633	0.94726
	1.00	2.67578	0.94726
	1.20	2.86524	0.94726
	1.40	3.05469	0.94727
	1.60	3.24414	0.94726
	1.80	3.43360	0.94726
	2.00	3.62305	0.94726
	2.20	3.81250	0.94727
	2.40	4.00195	0.94726
	2.60	4.19141	0.94727
	2.80	4.38086	0.94726
	3.00	4.57031	0.94727
	3.20	4.75977	0.94726
0.10	0.20	1.84229	
	0.40	2.03174	0.94726
	0.60	2.22120	0.94727
	0.80	2.41065	0.94726
	1.00	2.60010	0.94726
	1.20	2.78956	0.94726
	1.40	2.97901	0.94726
	1.60	3.16846	0.94726
	1.80	3.35791	0.94726
	2.00	3.54737	0.94726
	2.20	3.73682	0.94727
	2.40	3.92627	0.94726
	2.60	4.11573	0.94727
	2.80	4.30518	0.94726
	3.00	4.49463	0.94727
	3.20	4.68409	0.94726

Table B-22 Functional #1 with Payload Mass as Design Parameter - Horizontal trajectory

l_2	m_0	$\Sigma \lvert Wk_{in} \rvert$	Sensitivity
0.15	0.20	1.75798	
	0.40	1.94743	0.94726
	0.60	2.13688	0.94727
	0.80	2.32633	0.94726
	1.00	2.51579	0.94726
	1.20	2.70524	0.94726
	1.40	2.89469	0.94726
	1.60	3.08415	0.94726
	1.80	3.27360	0.94726
	2.00	3.46305	0.94726
	2.20	3.65251	0.94727
	2.40	3.84196	0.94726
	2.60	4.03141	0.94727
	2.80	4.22086	0.94726
	3.00	4.41032	0.94727
	3.20	4.59977	0.94726
0.20	0.20	1.66503	
	0.40	1.85448	0.94726
	0.60	2.04393	0.94727
	0.80	2.23338	0.94726
	1.00	2.42284	0.94726
	1.20	2.61229	0.94726
	1.40	2.80174	0.94727
	1.60	2.99120	0.94726
	1.80	3.18065	0.94726
	2.00	3.37010	0.94726
	2.20	3.55955	0.94727
	2.40	3.74901	0.94726
	2.60	3.93846	0.94726
	2.80	4.12791	0.94726
	3.00	4.31737	0.94727
	3.20	4.50682	0.94726

Table B-23 Functional #1 with Payload Mass as Design Parameter - Horizontal trajectory

l_2	m_0	$\Sigma \mid Wk_{in} \mid$	Sensitivity
0.25	0.20	1.56344	0.94726
	0.40	1.75289	0.94727
	0.60	1.94235	0.94727
	0.80	2.13180	0.94726
	1.00	2.32125	0.94726
	1.20	2.51070	0.94727
	1.40	2.70016	0.94726
	1.60	2.88961	0.94726
	1.80	3.07906	0.94727
	2.00	3.26852	0.94727
	2.20	3.45797	0.94727
	2.40	3.64742	0.94726
	2.60	3.83687	0.94727
	2.80	4.02633	0.94727
	3.00	4.21578	0.94726
	3.20	4.40523	0.94726

Table B-24 Functional #1 with Payload Mass as Design Parameter - Horizontal Trajectory

m_0	l_2	$\Sigma \lvert Wk_{in} \rvert$	Sensitivity
0.5	0.06	2.18771	
	0.08	2.15778	-1.49635
	0.10	2.12647	-1.56541
	0.12	2.09378	-1.63450
	0.14	2.05971	-1.70358
	0.16	2.02426	-1.77267
	0.18	1.98742	-1.84175
	0.20	1.94920	-1.91083
	0.22	1.90961	-1.97991
	0.24	1.86863	-2.04898
	0.26	1.82627	-2.11807
1.0	0.06	2.66134	
	0.08	2.63141	-1.49635
	0.10	2.60010	-1.56542
	0.12	2.56741	-1.63450
	0.14	2.53334	-1.70358
	0.16	2.49789	-1.77267
	0.18	2.46105	-1.84175
	0.20	2.42284	-1.91083
	0.22	2.38324	-1.97990
	0.24	2.34226	-2.04898
	0.26	2.29990	-2.11807
1.5	0.06	3.13497	
	0.08	3.10504	-1.49635
	0.10	3.07374	-1.56542
	0.12	3.04105	-1.63450
	0.14	3.00697	-1.70358
	0.16	2.97152	-1.77267
	0.18	2.93469	-1.84175
	0.20	2.89647	-1.91083
	0.22	2.85687	-1.97990
	0.24	2.81589	-2.04899
	0.26	2.77353	-2.11806

Table B-25 Functional #1 with Link l_2 as Design Parameter - Horizontal Trajectory

m_0	l_2	$\Sigma \lvert Wk_{in} \rvert$	Sensitivity
2.0	0.06	3.60860	
	0.08	3.57868	-1.49635
	0.10	3.54737	-1.56542
	0.12	3.51468	-1.63451
	0.14	3.48061	-1.70358
	0.16	3.44515	-1.77265
	0.18	3.40832	-1.84175
	0.20	3.37010	-1.91083
	0.22	3.33050	-1.97990
	0.24	3.28952	-2.04899
	0.26	3.24716	-2.11806
2.5	0.06	4.08224	
	0.08	4.05231	-1.49634
	0.10	4.02100	-1.56543
	0.12	3.98831	-1.63450
	0.14	3.95424	-1.70360
	0.16	3.91878	-1.77265
	0.18	3.88195	-1.84175
	0.20	3.84373	-1.91083
	0.22	3.80414	-1.97991
	0.24	3.76316	-2.04898
	0.26	3.72079	-2.11806
3.0	0.06	4.55587	
	0.08	4.52594	-1.49634
	0.10	4.49463	-1.56543
	0.12	4.46194	-1.63450
	0.14	4.42787	-1.70360
	0.16	4.39242	-1.77267
	0.18	4.35558	-1.84174
	0.20	4.31737	-1.91083
	0.22	4.27777	-1.97990
	0.24	4.23679	-2.04899
	0.26	4.19443	-2.11806

Table B-26 Functional #1 with Link l_2 as Design Parameter - Horizontal Trajectory

l_2	t_f	m_0	max $\lvert\tau_1\rvert$	sensitivity
0.05	0.44	0.2	11.14786	
		0.4	11.85532	3.53734
		0.6	12.56279	3.53734
		0.8	13.27026	3.53734
		1.0	13.97773	3.53734
		1.2	14.68520	3.53734
		1.4	15.39266	3.53734
		1.6	16.10013	3.53734
		1.8	16.80760	3.53734
		2.0	17.51507	3.53734
		2.2	18.22254	3.53734
		2.4	18.93000	3.53734
		2.6	19.63747	3.53735
		2.8	20.34494	3.53734
		3.0	21.05241	3.53733
		3.2	21.75988	3.53734
0.10	0.44	0.2	11.14786	
		0.4	11.85532	3.53734
		0.6	12.56279	3.53734
		0.8	13.27026	3.53734
		1.0	13.97773	3.53734
		1.2	14.68520	3.53734
		1.4	15.39266	3.53734
		1.6	16.10013	3.53734
		1.8	16.80760	3.53734
		2.0	17.51507	3.53734
		2.2	18.22254	3.53734
		2.4	18.93000	3.53734
		2.6	19.63747	3.53735
		2.8	20.34494	3.53734
		3.0	21.05241	3.53733
		3.2	21.75988	3.53734

Table B-27 Functional #2 with Payload Mass as Design Parameter - Horizontal Trajectory

l_2	t_f	m_0	max $\|\tau_1\|$	sensitivity
0.15	0.44	0.2	11.14786	
		0.4	11.85532	3.53734
		0.6	12.56279	3.53734
		0.8	13.27026	3.53734
		1.0	13.97773	3.53734
		1.2	14.68520	3.53734
		1.4	15.39266	3.53734
		1.6	16.10013	3.53734
		1.8	16.80760	3.53734
		2.0	17.51507	3.53734
		2.2	18.22254	3.53734
		2.4	18.93000	3.53734
		2.6	19.63747	3.53735
		2.8	20.34494	3.53734
		3.0	21.05241	3.53733
		3.2	21.75988	3.53734
0.20	0.44	0.2	11.14786	
		0.4	11.85532	3.53734
		0.6	12.56279	3.53734
		0.8	13.27026	3.53734
		1.0	13.97773	3.53734
		1.2	14.68520	3.53734
		1.4	15.39266	3.53734
		1.6	16.10013	3.53734
		1.8	16.80760	3.53734
		2.0	17.51507	3.53734
		2.2	18.22254	3.53734
		2.4	18.93000	3.53734
		2.6	19.63747	3.53735
		2.8	20.34494	3.53734
		3.0	21.05241	3.53733
		3.2	21.75988	3.53734

Table B-28 Functional #2 with Payload Mass as Design Parameter - Horizontal Trajectory

l_2	t_f	m_0	max $\|\tau_1\|$	sensitivity
0.25	0.44	0.2	11.14786	
		0.4	11.85532	3.53734
		0.6	12.56279	3.53734
		0.8	13.27026	3.53734
		1.0	13.97773	3.53734
		1.2	14.68520	3.53734
		1.4	15.39266	3.53734
		1.6	16.10013	3.53734
		1.8	16.80760	3.53734
		2.0	17.51507	3.53734
		2.2	18.22254	3.53734
		2.4	18.93000	3.53734
		2.6	19.63747	3.53735
		2.8	20.34494	3.53734
		3.0	21.05241	3.53733
		3.2	21.75988	3.53734

Table B-29 Functional #2 with Payload Mass as Design Parameter - Horizontal Trajectory

m_0	t_f	l_2	max $\lvert\tau_1\rvert$	sensitivity
0.50	0.44	0.06	12.20906	
		0.08	12.20906	0.0
		0.10	12.20906	0.0
		0.12	12.20906	0.0
		0.14	12.20906	0.0
		0.16	12.20906	0.0
		0.18	12.20906	0.0
		0.20	12.20906	0.0
		0.22	12.20906	0.0
		0.24	12.20906	0.0
		0.26	12.20906	0.0
1.00	0.44	0.06	13.97773	
		0.08	13.97773	0.0
		0.10	13.97773	0.0
		0.12	13.97773	0.0
		0.14	13.97773	0.0
		0.16	13.97773	0.0
		0.18	13.97773	0.0
		0.20	13.97773	0.0
		0.22	13.97773	0.0
		0.24	13.97773	0.0
		0.26	13.97773	0.0
1.50	0.44	0.06	15.74640	
		0.08	15.74640	0.0
		0.10	15.74640	0.0
		0.12	15.74640	0.0
		0.14	15.74640	0.0
		0.16	15.74640	0.0
		0.18	15.74640	0.0
		0.20	15.74640	0.0
		0.22	15.74640	0.0
		0.24	15.74640	0.0
		0.26	15.74640	0.0

Table B-30 Functional #2 with Link l_2 as Design Parameter - Horizontal Trajectory

m_0	t_f	l_2	max $\|\tau_1\|$	sensitivity
2.00	0.44	0.06	17.5151	
		0.08	17.5151	0.0
		0.10	17.5151	0.0
		0.12	17.5151	0.0
		0.14	17.5151	0.0
		0.16	17.5151	0.0
		0.18	17.5151	0.0
		0.20	17.5151	0.0
		0.22	17.5151	0.0
		0.24	17.5151	0.0
		0.26	17.5151	0.0
2.50	0.44	0.06	19.2837	
		0.08	19.2837	0.0
		0.10	19.2837	0.0
		0.12	19.2837	0.0
		0.14	19.2837	0.0
		0.16	19.2837	0.0
		0.18	19.2837	0.0
		0.20	19.2837	0.0
		0.22	19.2837	0.0
		0.24	19.2837	0.0
		0.26	19.2837	0.0
3.00	0.44	0.06	21.0524	
		0.08	21.0524	0.0
		0.10	21.0524	0.0
		0.12	21.0524	0.0
		0.14	21.0524	0.0
		0.16	21.0524	0.0
		0.18	21.0524	0.0
		0.20	21.0524	0.0
		0.22	21.0524	0.0
		0.24	21.0524	0.0
		0.26	21.0524	0.0

Table B-31 Functional #2 with Link l_2 as Design Parameter - Horizontal Trajectory

l_2	m_0	t_f	max $\lvert\tau_1\rvert$	max $\lvert\tau_2\rvert$	τ_1 sensitivities	τ_2
0.05	0.75	0.20	13.09339	3.464655		
		0.22	13.09339	3.464656	0.00000	0.00001
		0.24	13.09339	3.464655	0.00000	-0.00001
		0.26	13.09339	3.464655	0.00000	0.00000
		0.28	13.09339	3.464655	0.00000	0.00000
		0.30	13.09339	3.464654	0.00000	-0.00005
		0.32	13.09339	3.464654	0.00000	0.00000
		0.34	13.09339	3.464654	0.00000	0.00000
		0.36	13.09339	3.464653	0.00000	-0.00004
		0.38	13.09339	3.464654	0.00000	0.00001
		0.40	13.09339	3.464653	0.00000	-0.00001
		0.42	13.09339	3.464653	0.00000	0.00000
		0.44	13.09339	3.464653	0.00000	0.00000
0.10	1.50	0.20	15.74640	4.876393		
		0.22	15.74640	4.876393	0.00000	0.00000
		0.24	15.74640	4.876393	0.00000	0.00000
		0.26	15.74640	4.876393	0.00000	0.00000
		0.28	15.74640	4.876393	0.00000	0.00000
		0.30	15.74640	4.876391	0.00000	-0.00007
		0.32	15.74640	4.876391	0.00000	0.00000
		0.34	15.74640	4.876391	0.00000	0.00000
		0.36	15.74640	4.876390	0.00000	-0.00007
		0.38	15.74640	4.876393	0.00000	0.00002
		0.40	15.74640	4.876393	0.00000	-0.00002
		0.42	15.74640	4.876393	0.00000	0.00000
		0.44	15.74640	4.876393	0.00000	0.00000

Table B-32 Functionals #2 and #3 with t_f as Design Parameter - Horizontal Trajectory

l_2	m_0	t_f	max $\|\tau_1\|$	max $\|\tau_2\|$	τ_1 sensitivities	τ_2
0.15	2.25	0.20	18.39940	6.244536		
		0.22	18.39940	6.244536	0.00000	0.00002
		0.24	18.39940	6.244536	0.00000	0.00000
		0.26	18.39940	6.244536	0.00000	-0.00002
		0.28	18.39940	6.244536	0.00000	0.00000
		0.30	18.39940	6.244534	0.00000	-0.00007
		0.32	18.39940	6.244534	0.00000	0.00000
		0.34	18.39940	6.244534	0.00000	-0.00002
		0.36	18.39940	6.244533	0.00000	-0.00007
		0.38	18.39940	6.244533	0.00000	0.00000
		0.40	18.39940	6.244533	0.00000	0.00000
		0.42	18.39940	6.244533	0.00000	0.00000
		0.44	18.39940	6.244533	0.00000	0.00000
0.20	3.00	0.20	21.05241	7.569085		
		0.22	21.05241	7.569086	0.00000	0.00002
		0.24	21.05241	7.569085	0.00000	-0.00002
		0.26	21.05241	7.569085	0.00000	0.00000
		0.28	21.05241	7.569085	0.00000	0.00000
		0.30	21.05241	7.569083	0.00000	-0.00010
		0.32	21.05241	7.569083	0.00000	0.00000
		0.34	21.05241	7.569083	0.00000	-0.00002
		0.36	21.05241	7.569081	0.00000	-0.00007
		0.38	21.05241	7.569081	0.00000	0.00000
		0.40	21.05241	7.569081	0.00000	-0.00002
		0.42	21.05241	7.569081	0.00000	0.00000
		0.44	21.05241	7.569081	0.00000	0.00000

Table B-33 Functionals #2 and #3 with t_f as Design Parameter - Horizontal Trajectory

l_2	t_f	m_0	max $\lvert\tau_2\rvert$	sensitivity
0.05	0.44	0.2	2.14919	
		0.4	2.62751	2.39175
		0.6	3.10589	2.39175
		0.8	3.58424	2.39175
		1.0	4.06259	2.39175
		1.2	4.54092	2.39175
		1.4	5.01929	2.39175
		1.6	5.49764	2.39175
		1.8	5.97599	2.39175
		2.0	6.45434	2.39175
		2.2	6.93269	2.39175
		2.4	7.41104	2.39175
		2.6	7.88939	2.39175
		2.8	8.36774	2.39175
		3.0	8.84609	2.39175
		3.2	9.32445	2.39176
0.10	0.44	0.2	1.76711	
		0.4	2.24546	2.39175
		0.6	2.72381	2.39175
		0.8	3.20216	2.39175
		1.0	3.68051	2.39175
		1.2	4.15887	2.39175
		1.4	4.63722	2.39175
		1.6	5.11557	2.39175
		1.8	5.59392	2.39175
		2.0	6.07227	2.39175
		2.2	6.55062	2.39175
		2.4	7.02897	2.39175
		2.6	7.50732	2.39175
		2.8	7.98567	2.39175
		3.0	8.46402	2.39175
		3.2	8.94237	2.39176

Table B-34 Function # 3 Values with Changes in Payload Mass - - Horizontal Trajectory

l_2	t_f	m_0	max $\lvert\tau_2\rvert$	sensitivity
0.15	0.44	0.2	1.34144	
		0.4	1.81979	2.39175
		0.6	2.29814	2.39175
		0.8	2.77649	2.39175
		1.0	3.25484	2.39175
		1.2	3.73319	2.39175
		1.4	4.21154	2.39175
		1.6	4.68989	2.39175
		1.8	5.16824	2.39175
		2.0	5.64659	2.39175
		2.2	6.12495	2.39175
		2.4	6.60330	2.39175
		2.6	7.08165	2.39176
		2.8	7.56000	2.39175
		3.0	8.03835	2.39175
		3.2	8.51670	2.39175
0.20	0.44	0.2	0.87218	
		0.4	1.35053	2.39175
		0.6	1.82888	2.39175
		0.8	2.30723	2.39175
		1.0	2.78558	2.39175
		1.2	3.26393	2.39175
		1.4	3.74228	2.39175
		1.6	4.22063	2.39175
		1.8	4.69898	2.39175
		2.0	5.17733	2.39175
		2.2	5.65568	2.39175
		2.4	6.13403	2.39175
		2.6	6.61238	2.39175
		2.8	7.09073	2.39175
		3.0	7.56908	2.39175
		3.2	8.04743	2.39175

Table B-35 Function # 3 Values with Changes in Payload Mass - - Horizontal Trajectory

l_2	t_f	m_0	max $\|\tau_2\|$	sensitivity
0.25	0.44	0.2	0.35932	
		0.4	0.83767	2.39175
		0.6	1.31602	2.39175
		0.8	1.79437	2.39175
		1.0	2.27272	2.39175
		1.2	2.75107	2.39175
		1.4	3.22942	2.39175
		1.6	3.70777	2.39175
		1.8	4.18612	2.39175
		2.0	4.66447	2.39175
		2.2	5.14282	2.39175
		2.4	5.62117	2.39175
		2.6	6.09952	2.39175
		2.8	6.57787	2.39175
		3.0	7.05622	2.39176
		3.2	7.53457	2.39175

Table B-36 Function # 3 Values with Changes in Payload Mass - Horizontal Trajectory

m_0	t_f	l_2	max $\|\tau_2\|$	sensitivity
0.50	0.44	0.06	2.79379	
		0.08	2.64270	- 7.5543
		0.10	2.48464	- 7.9031
		0.12	2.31960	- 8.2519
		0.14	2.14759	- 8.6006
		0.16	1.96860	- 8.9494
		0.18	1.78264	- 9.2981
		0.20	1.58970	- 9.6469
		0.22	1.38979	- 9.9956
		0.24	1.18290	-10.3444
		0.26	0.96904	-10.6931
1.00	0.44	0.06	3.98966	
		0.08	3.83858	- 7.5543
		0.10	3.68051	- 7.9031
		0.12	3.51548	- 8.2519
		0.14	3.34346	- 8.6006
		0.16	3.16448	- 8.9494
		0.18	2.97852	- 9.2981
		0.20	2.78558	- 9.6469
		0.22	2.58567	- 9.9956
		0.24	2.37878	-10.3444
		0.26	2.16492	-10.6931
1.50	0.44	0.06	5.18554	
		0.08	5.03445	- 7.5543
		0.10	4.87639	- 7.9031
		0.12	4.71135	- 8.2519
		0.14	4.53934	- 8.6006
		0.16	4.36035	- 8.9494
		0.18	4.17439	- 9.2981
		0.20	3.98145	- 9.6469
		0.22	3.78154	- 9.9956
		0.24	3.57465	-10.3444
		0.26	3.36079	-10.6931

Table B-37 Function #3 Calculations as Length of Link #2 Increases - Horizontal Trajectory

m_0	t_f	l_2	max $\lvert\tau_2\rvert$	sensitivity
2.00	0.44	0.06	6.38141	
		0.08	6.23033	- 7.5543
		0.10	6.07227	- 7.9031
		0.12	5.90723	- 8.2519
		0.14	5.73522	- 8.6006
		0.16	5.55623	- 8.9494
		0.18	5.37028	- 9.2981
		0.20	5.17733	- 9.6469
		0.22	4.97742	- 9.9956
		0.24	4.77053	-10.3444
		0.26	4.55668	-10.6931
2.50	0.44	0.06	7.57729	
		0.08	7.42620	- 7.5543
		0.10	7.26814	- 7.9031
		0.12	7.10310	- 8.2519
		0.14	6.93109	- 8.6006
		0.16	6.75210	- 8.9494
		0.18	6.56614	- 9.2981
		0.20	6.37320	- 9.6469
		0.22	6.17329	- 9.9956
		0.24	5.96640	-10.3444
		0.26	5.75254	-10.6931
3.00	0.44	0.06	8.77317	
		0.08	8.62208	- 7.5543
		0.10	8.46402	- 7.9031
		0.12	8.29898	- 8.2519
		0.14	8.12697	- 8.6006
		0.16	7.94798	- 8.9494
		0.18	7.76202	- 9.2981
		0.20	7.56908	- 9.6469
		0.22	7.36917	- 9.9956
		0.24	7.16228	-10.3444
		0.26	6.94842	-10.6931

Table B-38 Function #3 Calculations as Length of Link #2 Increases - Horizontal Trajectory

Bibliography

1. Bandler, John W. and others. "A Nonlinear l_1 Optimization Algorithm for Design, Modeling, and Diagnosis of Networks," IEEE Transactions on Circuits and Systems, 2: 174 - 181 (February 1987).

2. Hertzberg, H.T.E. "Engineering Anthropology," Human Engineering Guide to Equipment Design. Washington D.C.: U.S. Government Printing Office, 1972.

3. Kanoh, H. and others. "Modelling and Control of Flexible Robot Arms," Proceedings of 25th Conference on Decision and Control. 1866-1870. New York: IEEE Press, 1986.

4. Komkov, Vadim. "Control and Optimization Schemes in Optimal Design of Engineering Systems," National Science Foundation Grant # DMC - 8416155. 1-34.

5. Komkov, Vadim and V. Dannon. "Design Optimization for Structural or Mechanical Systems Against the Worst Possible Loads," Acta Applicandae Mathematicae 8: 37-64. (1987)

6. Komkov, Vadim and Carlton Irwin. "Uniqueness For Gradient Methods In Engineering Optimization," Proceedings of a Special Session of the American Mathematical Society. Berlin Heidelberg New York Tokyo: Springer-Verlag, 1984.

7. Koo, Delia. Elements of Optimization. New York: Springer-Verlag, 1977.

8. Kunz, Kaiser S. Numerical Analysis, New York: McGraw Hill, 1957.

9. Parker, Capt Steven L. Configuration Comparison Analysis For the AFIT/AAMRL Anthropomorphic Robotic Manipulator MS theses, AFIT/GA/AA/88D-08. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB, Ohio, December 1988.

10. Rivin, E.I., Mechanical Design of Robots, New York: McGraw Hill Book Company, 1988.

11. Spenny C. H. and M. B. Leahy. "Metrics For Choosing The Kinematic Structure of a Manipulator." Submitted to ASME Transactions on Dynamic Measurement and Control.

12. Vincent, Thomas L. and Walter J. Grantham. Optimality in Parametric Systems. New York: John Wiley & Sons, 1981.

Vita

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Lt Tumlin is married to the former Cynthia A. Payne, and has two children, Jillian and Carrie.

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Abstract

A method to optimize a robotic parallel manipulator configuration using Pareto Optimization techniques was developed. Pareto optimization is a cooperative effort between design parameters. The design parameters to be optimized included the payload mass, the length of the manipulator link labelled l_2, and the prescribed time for the manipulator to move a prescribed distance. Three functionals were computed for design optimization. These included the mechanical efficiency of the system, the maximum value of torque for motor one, and the maximum value of torque for motor two.

A functional analysis was performed using two trajectories for the manipulator; a horizontal trajectory and a vertical trajectory. A combination of these paths allow the manipulator to reach anywhere within its workspace.

Algorithms were developed for computing each of the functionals when changing any of the design parameters. When the horizontal path was traversed, mechanical efficiency was zero, thus total input work of the manipulator was evaluated. The sensitivities of the design parameter changes were evaluated for optimization. When a horizontal path was followed, only the link (l_2) length had changing sensitivity values. Sensitivity changes occurred for all of the design parameters for a vertical trajectory. Theses.